2.3.25) Suppose that $AB$ is nonsingular. By Theorem 2.9, the only solution to $ABx = 0$ is $x = 0$. If $B$ were singular, then there would be some $y \neq 0$ such that $By = 0$, in which case, we would have $ABy = A(By) = A0 = 0$, which gives us a nontrivial solution to $ABx = 0$, meaning that $AB$ would be singular. Therefore, $B$ is nonsingular.

If $A$ were singular, then we would have some $y \neq 0$ such that $Ay = 0$. Since $B$ is nonsingular, there is a unique solution $x = z$ to $Bx = y$. Furthermore, we have $z \neq 0$, as $B0 = 0 \neq y$. For this $z$ we get $ABz = Ay = 0$, so this is a nontrivial solution to $ABx = 0$, meaning again that $AB$ is singular. Hence, $A$ cannot be singular, either.

This problem would be much easier if we had Theorems 3.8 and 3.9 available. With those theorems, $AB$ is nonsingular if and only if $\det(AB) \neq 0$. Since $\det(AB) = \det(A) \det(B)$, this happens if and only if $\det(A) \neq 0$ and $\det(B) \neq 0$. This occurs if and only if both $A$ and $B$ are nonsingular.