Most of these problems appeared on a prelim a previous time that I taught this course. The exception is #4, which I came up with when trying to devise a proof problem for this prelim. I rejected it as being too hard, but give it to you as a practice problem anyway. On the older prelims, problems 5-8 here were worth only half as many points as problems 1-3 here.

1. Solve the following system of equations:
\[ \begin{align*}
 a + 3b - 2c - 2d &= 11 \\
-a - 2b + 4c + 5d &= -4 \\
-2a - 4b + 9c + 11d &= -7 
\end{align*} \]

2. Determine whether the following matrix is invertible, and if so, find its inverse:
\[
\begin{bmatrix}
 2 & 5 & 3 \\
 3 & 7 & 4 \\
 5 & 2 & -2 
\end{bmatrix}
\]

3. Let \[\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}^{-1} X \begin{bmatrix} -1 & 0 \\ 5 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & -2 \\ 3 & 0 \end{bmatrix} \]. Solve for the matrix \(X\).

4. Let \(A\) be a square matrix whose entries are all integers. Show that \(\text{det}(A)\) is also an integer.

5. Three of these systems of linear equations have the same set of solutions. Determine the one that differs and justify your answer.
\[
\begin{align*}
\begin{bmatrix}
 2 & 4 & 1 & -3 \\
 0 & 3 & -5 & 7 \\
 0 & 0 & 0 & -4 
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 
\end{bmatrix} &= \begin{bmatrix} 0 
\end{bmatrix} \\
\begin{bmatrix}
 2 & 1 & 6 & -10 \\
 0 & 3 & -5 & -1 \\
 0 & 0 & 0 & 2 
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 
\end{bmatrix} &= \begin{bmatrix} 0 
\end{bmatrix} \\
\begin{bmatrix}
 4 & 5 & -3 & 1 \\
 0 & 3 & -5 & 7 \\
 0 & 0 & 3 & 2 
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 
\end{bmatrix} &= \begin{bmatrix} 0 
\end{bmatrix} \\
\begin{bmatrix}
 4 & 5 & -3 & 1 \\
 0 & 6 & -10 & 6 \\
 0 & 0 & 0 & 1 
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 
\end{bmatrix} &= \begin{bmatrix} 0 
\end{bmatrix}
\end{align*}
\]

6. Exactly one of these matrices is invertible. Which one? Be sure to justify your answer.
\[
\begin{bmatrix}
 2 & 3 & 1 & 5 \\
 3 & 9 & 0 & -1 \\
 4 & -3 & -1 & 2 
\end{bmatrix} \quad \begin{bmatrix}
 3 & 2 & 5 \\
 0 & 4 & 1 \\
 0 & 0 & 0 
\end{bmatrix} \quad \begin{bmatrix}
 2 & 3 & 0 \\
 4 & -1 & -5 \\
 0 & 6 & 2 
\end{bmatrix}
\]
7. Let $A$ be a $4 \times 4$ matrix with $\det A = 4$. Compute $\det(A^2)$, $\det(2A)$, and $\det(2(A^{-1}))$.

8. Find the determinant of the matrix

$$
\begin{bmatrix}
2 & 3 & 1 & -2 & 4 & 0 \\
5 & -4 & 5 & 3 & -2 & 1 \\
-3 & -4 & -1 & 2 & 0 & 5 \\
5 & -4 & 5 & 3 & -2 & 1 \\
6 & 2 & 1 & 1 & -4 & -2 \\
3 & 5 & -3 & -2 & 0 & 1
\end{bmatrix}
$$