MATH 231

Linear Algebra with Applications

Fall 2006

Prelim 2 Solution

TA: Yu Tung Cheng

Mean = 20.06
Standard Deviation = 6.91

Problem 1.

(7 points, one point each)
a. Determine whether the set S is a subspace in V in each of the following cases:

1. $S = \{(x_1, x_2)^T \,|\, x_1 x_2 = 0\}\,,\ V = \mathbf{R}^2\,;$
   Solution: No. Addition is not closed.

2. $S = \{(x_1, x_2, x_3)^T \,|\, x_1 = x_2 = x_3\}\,,\ V = \mathbf{R}^3\,;$
   Solution: Yes, check all axioms.

3. S is the set of all symmetric 3×3 matrices in $V = \mathbf{M}_{3\times 3}(\mathbf{R})\,;$
   Solution: Yes, check all axioms.

4. S is the set of all nonsingular $n \times n$ matrices in $V = \mathbf{M}_{n\times n}(\mathbf{R})\,.$
   Solution: No. Zero matrix is not in S.

b. Let V be the space of all polynomials of degree $< 3\,.$ Find which of the following are spanning sets for $V\,:$
$(i)\,\{1, x, x-1, x-2\}\,,$
Solution: No. x^2 cannot be formed.
$(ii)\,\{x^2-1, x-2, x+2\}\,,$
Solution: Yes.
$(iii)\,\{2, x^2, x, 5x+2\}\,.$
Solution: Yes.

Problem 2.

(3 points)
Determine whether the following vectors are linearly independent:
$(i)\,\{(1,2)^T, (-1,1)^T, (-2,1)^T\}$ in $\mathbf{R}^2\,,$
Solution: No.
$(ii)\,\{(1,2,0)^T, (-1,0,1)^T, (-1,1,1)^T\}$ in $\mathbf{R}^3\,,$

Solution: Yes.
(*iii*) $\{1, x, x-1, x^2-2\}$ in P_3.
Solution: No.

Problem 3.

(8 points) **a**. In Problem 1(**a**) above find $\dim(S)$ in each case when S *is* a subspace.

b. In Problems 1(**b**) and 2 above determine whether the indicated sets of vectors are bases.

(a) 2. 1 3. 6
(b) 1b(ii), 2(ii)

Problem 4.

(12 points) Let $S = \{(1,1,1)^T, (2,3,2)^T, (1,5,4)^T\}$ and $T = \{(1,1,0)^T, (1,2,0)^T, (1,2,1)^T\}$ be two ordered bases in $\mathbf{R}^3$. Find the transition matrix from S to T. If $\underline{u}$ has coordinates $(3,2,-1)$ and $\underline{v}$ has coordinates $(1,-3,2)$ with respect to the basis S, what are the coordinates of $\underline{u}$ and $\underline{v}$ with respect to T?

$$\begin{pmatrix} 1 & 1 & -3 \\ -1 & -1 & 0 \\ 1 & 2 & 4 \end{pmatrix}, \begin{pmatrix} 8 \\ -5 \\ 3 \end{pmatrix}, \begin{pmatrix} -8 \\ 2 \\ 3 \end{pmatrix}$$