5. (h) $a\begin{bmatrix}-1\\1\\7\end{bmatrix}+2\begin{bmatrix}a\\a\\0\end{bmatrix}-4\begin{bmatrix}0\\-1\\a\end{bmatrix}=\begin{bmatrix}a\\3a+4\\3a\end{bmatrix}$

6. (c) Since $2u-3v+5w=\begin{bmatrix}2a\\-10\end{bmatrix}-\begin{bmatrix}3\\18-b\end{bmatrix}+\begin{bmatrix}15\\5\end{bmatrix}=\begin{bmatrix}2a+12\\3b-23\end{bmatrix}$, we must have $2a+12=0$ and $3b-23=0$, so $a=-6$, $b=\frac{23}{3}$.

11. (a) [BB] The typical linear combination of $\begin{bmatrix}-2\\-2\end{bmatrix}$ and $\begin{bmatrix}3\\3\end{bmatrix}$ is a vector of the form $a\begin{bmatrix}-2\\-2\end{bmatrix}+b\begin{bmatrix}3\\3\end{bmatrix}=\begin{bmatrix}-2a+3b\\-2a+3b\end{bmatrix}$. This is a vector both of whose components are equal, so it can never be $\begin{bmatrix}1\\2\end{bmatrix}$. The answer is "no".

(b) 1.1.14 says that every vector is a linear combination of two two-dimensional vectors **which are not parallel**. The vectors in part (a) are parallel, however—$\begin{bmatrix}-2\\-2\end{bmatrix}=-\frac{2}{3}\begin{bmatrix}3\\3\end{bmatrix}$—so there is no contradiction.

14. (a) [BB] $u=\begin{bmatrix}2\\2\end{bmatrix}$; $v=\begin{bmatrix}2\\-1\end{bmatrix}$.

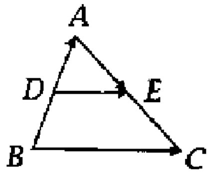
(b) (a) [BB] $\overrightarrow{OD}=\begin{bmatrix}4\\1\end{bmatrix}=u+v$ (b) $\overrightarrow{OG}=\begin{bmatrix}6\\0\end{bmatrix}=u+2v$

(c) $\overrightarrow{OH}=\begin{bmatrix}8\\2\end{bmatrix}=2u+2v$ (d) $\overrightarrow{CA}=\begin{bmatrix}0\\3\end{bmatrix}=u-v$

(e) $\overrightarrow{AF}=\begin{bmatrix}2\\-4\end{bmatrix}=-u+2v$ (f) $\overrightarrow{GE}=\begin{bmatrix}0\\3\end{bmatrix}=u-v$

(g) $\overrightarrow{HC}=\begin{bmatrix}-6\\-3\end{bmatrix}=-2u-v$

15. We wish to show that $\overrightarrow{DE}=\frac{1}{2}\overrightarrow{BC}$. Now $\overrightarrow{AD}=\frac{1}{2}\overrightarrow{AB}$ and $\overrightarrow{AE}=\frac{1}{2}\overrightarrow{AC}$, so $\overrightarrow{DE}=\overrightarrow{DA}+\overrightarrow{AE}=-\frac{1}{2}\overrightarrow{AB}+\frac{1}{2}\overrightarrow{AC}=\frac{1}{2}(\overrightarrow{AC}-\overrightarrow{AB})=\frac{1}{2}\overrightarrow{BC}$.



1.2

3. (a) [BB] Since $u \cdot v = 0$, the vectors are orthogonal: $\theta = \frac{\pi}{2}$.

   (d) $u \cdot v = 2 - 1 + 2 = 3$, $\|u\| = \sqrt{6} = \|v\|$, so $\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|} = \dfrac{3}{6} = \dfrac{1}{2}$. $\theta = \arccos(\frac{1}{2}) = \frac{\pi}{3}$.

7. (a) [BB] $\|u\| = \sqrt{3^2 + 4^2 + 0^2} = 5$; $u + v = \begin{bmatrix} 5 \\ 5 \\ 2 \end{bmatrix}$, so $\|u + v\| = \sqrt{5^2 + 5^2 + 2^2} = \sqrt{54}$; $\left\| \frac{w}{\|w\|} \right\| = 1$ (this is true for any $w \neq 0$).

   (b) [BB] The answer is $\frac{1}{\|u\|} u = \frac{1}{5} u = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \\ 0 \end{bmatrix}$.

   (c) [BB] First we find a vector of norm 1 in the direction of v; this vector is $\frac{1}{\|v\|} v = \frac{1}{3} v$. A vector of norm 4 in this direction is $\frac{4}{3} v$, so a vector of norm 4 in the opposite direction is $-\frac{4}{3} v = \begin{bmatrix} -\frac{8}{3} \\ -\frac{4}{3} \\ -\frac{8}{3} \end{bmatrix}$.

9. We are given $u \cdot u = 3^2 = 9$ and $v \cdot v = 5^2 = 25$.

   (b) $(-3u + 4v) \cdot (2u + 5v) = -6u \cdot u - 7u \cdot v + 20v \cdot v = -6(9) - 7(8) + 20(25) = 390$.

   (f) [BB] $\|u + v\|^2 = (u + v) \cdot (u + v) = u \cdot u + 2u \cdot v + v \cdot v = 9 + 2(8) + 25 = 50$.

23. (a) $\|u + v\|^2 = (u+v) \cdot (u+v) = u \cdot u + 2u \cdot v + v \cdot v = \|u\|^2 + 2u \cdot v + \|v\|^2 \leq \|u\|^2 + 2|u \cdot v| + \|v\|^2 \leq \|u\|^2 + 2\,\|u\|\,\|v\| + \|v\|^2 = (\|u\| + \|v\|)^2$. The result follows upon taking the square root of each side.

28. Let θ be the angle between u and v. If $u \cdot v = 9$, $\|u\| = 5$ and $\|v\| = 1$, then $\cos\theta = \frac{u \cdot v}{\|u\|\|v\|} = \frac{9}{5}$. This is impossible since $-1 \leq \cos\theta \leq +1$ for any θ.

32. We want $(u + kv) \cdot u = 0$; that is, $u \cdot u + kv \cdot u = 0$. Now $v \cdot u \neq 0$ since u and v are not orthogonal, so $k = -\dfrac{u \cdot u}{v \cdot u}$.

3. (b) The answer is an equation of the form $18x+6y-5z=d$. Substituting $x=-1$, $y=1$, $z=7$, we get $d=-18+6-35=-47$, so the plane has equation $18x+6y-5z=-47$.

6. Let θ be the angle between u and v. If $\|\mathsf{u}\times\mathsf{v}\|=15$, $\|\mathsf{u}\|=3$ and $\|\mathsf{v}\|=4$, then $\sin\theta=\frac{\|\mathsf{u}\times\mathsf{v}\|}{\|\mathsf{u}\|\|\mathsf{v}\|}=\frac{15}{12}$. This is impossible since the sine of an angle cannot be greater than 1.

8. $\begin{vmatrix} \mathsf{i} & \mathsf{j} & \mathsf{k} \\ 4 & 3 & -1 \\ 2 & 3 & 0 \end{vmatrix} = 3\mathsf{i}-2\mathsf{j}+6\mathsf{k} = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$ is a vector n perpendicular to both the given vectors.

Since $\|\mathsf{n}\|=\sqrt{3^2+(-2)^2+6^2}=\sqrt{49}=7$, one possible answer is $\frac{1}{7}\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$. The other possibility is $-\frac{1}{7}\begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$.

9. [BB] Call the given vectors $\mathsf{u}=\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$ and $\mathsf{v}=\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}$. One vector perpendicular to both u and v is

$$\mathsf{u}\times\mathsf{v}=\begin{vmatrix} \mathsf{i} & \mathsf{j} & \mathsf{k} \\ 1 & 3 & -1 \\ 5 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 3 & -1 \\ 0 & 1 \end{vmatrix}\mathsf{i} - \begin{vmatrix} 1 & -1 \\ 5 & 1 \end{vmatrix}\mathsf{j} + \begin{vmatrix} 1 & 3 \\ 5 & 0 \end{vmatrix}\mathsf{k}$$

$$= 3\mathsf{i}-6\mathsf{j}-15\mathsf{k} = \begin{bmatrix} 3 \\ -6 \\ -15 \end{bmatrix}.$$

11 (b) $\overrightarrow{AB}=\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\overrightarrow{AC}=\begin{bmatrix} 8 \\ -5 \\ -1 \end{bmatrix}$. A normal vector is $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix} \mathsf{i} & \mathsf{j} & \mathsf{k} \\ 1 & -1 & 0 \\ 8 & -5 & -1 \end{vmatrix}=$ $1\mathsf{i}-(-1)\mathsf{j}+3\mathsf{k}=\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$. The plane has equation of the form $x+y+3z=d$. Since the coordinates of A satisfy the equation, we have $-1+2+3=d$, so $d=4$ and the equation is $x+y+3z=4$.

15. (a) [BB] We need triples (x,y,z) that satisfy both equations. Setting $x=0$ gives $y+z=5$, $-y+z=1$, so $z=3$ and $y=2$. This gives the point $A(0,2,3)$. Setting $y=0$ gives $2x+z=5$, $x+z=1$, so $x=4$ and $z=-3$. This gives the point $B(4,0,-3)$. Setting $z=0$ gives $2x+y=5$, $x-y=1$, so $x=2$ and $y=1$. This gives the point $C(2,1,0)$. Many other points are possible, of course.