Math 2240 Problem Set 1

Due: January 29th, 2013 in class

When you hand in this problem set, please indicate on the top of the front page how much time it took you to complete.

Reading: 4.1–4.3

Problems from the book:
4.1.2, 4.1.6, 4.1.12, 4.1.14, 4.1.17, 4.1.18, 4.1.20.
4.2.1, 4.2.2, 4.2.6, 4.2.7.

Additional problems:

1. Suppose that a meter stick is broken at two randomly chosen points. What is the probability that the three segments form the sides of a triangle?

2. Let \( f(x) : [0, 1] \to \mathbb{R} \) be a function. The total variation of function \( f \) is the quantity

\[
V(f) = \sup_P \sum_{i=0}^{n-1} |f(x_{i+1}) - f(x_i)|.
\]

where the supremum is taken over the set all partitions

\[ P = \{0 = x_0 < x_1 < \cdots < x_{n_P} = 1\} \]

of the interval \([0, 1]\).

The function \( f \) has bounded variation if \( V(f) < \infty \).

(a) Show that every monotone function has bounded variation.

(b) Prove that if \( f \) has bounded variation then \( f \) is integrable.

(c) Construct an example of integrable function which does not have bounded variation.