Questions from the book

Section 4.10 - Questions 4, 10, 12, 13, 16
Section 4.11 - Questions 2, 4, 11, 15, 16

Additional problem

The Sobolev representation formula

Prove the Sobolev representation formula in $\mathbb{R}^2$ that expresses $f$ as an integral involving $\frac{\partial f}{\partial x_1}$ and $\frac{\partial f}{\partial x_2}$ (so it is a generalisation of the fundamental theorem of the calculus). The hypothesis is that $f$ is $C^1$ and has compact support, and the formula says,

$$f(x) = c \left( \int_{\mathbb{R}^2} \frac{\partial f}{\partial x_1}(x - y) \frac{y_1}{|y|^2} dy + \int_{\mathbb{R}^2} \frac{\partial f}{\partial x_2}(x - y) \frac{y_2}{|y|^2} dy \right)$$

for a specific constant $c$ (part of the problem is to figure out what $c$ is).

Note that these are convolutions. Part of the problem is to verify that these integrals exist in the sense of Lebesgue.

**Hint:** Start with the fundamental theorem of the calculus

$$f(x) = -\int_0^\infty \frac{d}{dr} \left( f(x - r \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}) \right) dr$$

and integrate with respect to $\theta$. 