Math 2210 - Linear Algebra
Second prelim - 5 November 2013 - 7:30 to 9:00pm

Name and NetID: ________________________________

Whose discussion section are you enrolled in? Circle one. Yash Lodha  Wai-kit Yeung
At what time is the discussion section you enrolled in? Circle one.
1:25-2:15pm  2:30-3:20pm  3:35-4:25pm

INSTRUCTIONS

• This test has 6 problems on 6 pages, worth a total of 100 points. Check if you have all 6 pages with questions.

• If you need more space than you are given under a question, clearly indicate where the remaining work is and point out where your final answer is. You also have a 2-sided page for scratchwork at the end.

• No books or electronic devices allowed. You are allowed a one-sided letter size paper of notes.

• Please show all your work and justify your answers.

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1. _____ / 15
2. _____ / 20
3. _____ / 15
4. _____ / 20
5. _____ / 10
6. _____ / 20

Total: _____ / 100

Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty.
Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.
Please sign below to indicate that you have read and agree to these instructions.

Signature of Student
1. (15 points) Let

\[ A = \begin{pmatrix} -2 & 1 & -4 & -3 \\ 0 & 0 & -1 & -1 \\ 2 & 1 & 5 & 3 \end{pmatrix}. \]

Compute a basis for each of the following spaces:

(a) The column space of A.
(b) The row space of A.
(c) The null space of A.
2. (20 points) Let

\[ A = \begin{pmatrix} 2 & -3 & -3 \\ 0 & 2 & 0 \\ 0 & -1 & 1 \end{pmatrix}. \]

Is \( A \) diagonalizable? If not, justify. If so, find an invertible matrix \( P \) and a diagonal matrix \( D \) such that \( A = PDP^{-1} \).
3. (15 points) Let

\[ A = \begin{pmatrix} -1 & -6 & 2 \\ -1 & -9 & 3 \\ 2 & 20 & -7 \end{pmatrix}. \]

(a) Calculate the determinant of \( A \).

(b) Let \( \vec{r}_1 \), \( \vec{r}_2 \) and \( \vec{r}_3 \) be the 3 rows of \( A \). What is the volume of the parallelepiped spanned by \( \vec{r}_1 \), \( 2\vec{r}_2 \) and \( 3\vec{r}_3 \)?

(c) Consider the linear transformation \( T(\vec{x}) = A\vec{x} \). What is the volume of the parallelepiped spanned by \( T(\vec{r}_1) \), \( T(\vec{r}_2) \) and \( T(\vec{r}_3) \)?
4. \(20 \text{ points}\) The space \(F\) of all functions \(\mathbb{R}^3 \rightarrow \mathbb{R}\) is a vector space. Consider the subset \(L\) of linear transformations \(\mathbb{R}^3 \rightarrow \mathbb{R}\).

(a) Show that \(L\) is a vector subspace.

(b) Compute a basis for \(L\).

(c) Find the coordinates of \(T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = x + y + z\) in the basis from part (b).
5. (10 points) Suppose that you are working with a large set of 53 homogeneous equations in 56 variables. If you found 3 linearly independent solutions to this system such that any other solution is in the span of these three, can you be certain that the nonhomogenous system has a solution? Justify your answer.
6. (20 points) For each of the following statements, either prove the statement is true or provide a counter example to show it is false.

(a) If $A$ and $B$ are similar matrices then they have the same eigenvalues.

(b) If $A$, $P$ and $Q$ are $2 \times 2$ matrices such that

$$
\begin{pmatrix}
1 & 0 \\
0 & 0
\end{pmatrix} = P^{-1}AP = Q^{-1}AQ,
$$

then $P$ and $Q$ are scalar multiples of each other.

(c) There is no $3 \times 3$ matrix $A$ such that both $A$ and $A - I_3$ have null spaces of dimension 2.

(d) Given a matrix $A$ and a vector $\vec{x}$, if $A\vec{x}$ is an eigenvector of $A$ then $\vec{x}$ is an eigenvector for $A$. 


This page is for scratch work, it will not be graded unless you point us here from the page where the question was posed.