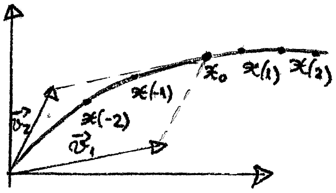
HW 11

$\boxed{7.1}$ $\boxed{28}$ $\lambda_1 = 1.2$, $\lambda_2 = 1.1$



$\boxed{34}$ $A\vec{v} = \lambda\vec{v} = 4\vec{v}$

$$(A^2 + 2A + 3I_n)\vec{v} = \lambda^2\vec{v} + 2\lambda\vec{v} + 3\vec{v} = (\lambda^2 + 2\lambda + 3)\vec{v}$$

Since $A^n\vec{v} = \lambda^n\vec{v}$

$\Rightarrow$ The evalue is $(\lambda^2 + 2\lambda + 3) = 27$

$\boxed{7.3}$ $\boxed{32}$

We know $\text{rank}(A) = \text{rank}(A^T)$, for any A, hence

$$\text{rank}(A - \lambda I) = \text{rank}((A - \lambda I)^T) = \text{rank}(A^T - \lambda I) \Rightarrow$$

By rank-nullity $\Rightarrow$

$$\text{nullity}(A - \lambda I) = \text{nullity}(A^T - \lambda I)$$

$\Leftrightarrow$ geom. mult (λ_A) = geom. mult (λ_{A^T})

$\boxed{7.2}$ $\boxed{8}$ $c_A(x) = \det(A - xI) = -x^2(x+3) \Rightarrow \begin{cases} \lambda_1 = 0 \text{ with alg. mult. } 2 \\ \lambda_2 = -3 \text{ with alg. mult. } 1 \end{cases}$

$\boxed{12}$ $c_A(x) = x(x-1)^2(x+1) \Rightarrow \begin{cases} \lambda_1 = 0 \text{ - alg. mult. } 1 \\ \lambda_2 = 1 \text{ - alg. mult. } 2 \\ \lambda_3 = -1 \text{ - alg. mult. } 1 \end{cases}$

$\boxed{18}$ $c_A(x) = x^2 - 2ax + a^2 - b^2 = [x - (a+b)][x - (a-b)] \Rightarrow \begin{cases} \lambda_1 = a + b \\ \lambda_2 = a - b \end{cases}$

$\boxed{38}$ $c_A(x) = x^2 - \text{tr}(A)\cdot x + \det A = x^2 - 5x - 14 = (x+2)(x-7) \Rightarrow \begin{cases} \lambda_1 = -2 \\ \lambda_2 = 7 \end{cases}$

$\boxed{7.3}$ $\boxed{14}$ $c_A(x) = -x(x-1)^2 \Rightarrow \begin{array}{l} \lambda_1 = 0, \text{ alg. mult. } 1 \\ \lambda_2 = 1, \text{ alg. mult. } 2 \end{array}$

$$\left.\begin{array}{l} E_0 = \ker A = \text{Span}\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \lambda_1 = 0 \text{ - geom. mult. } 1 \\ E_1 = \ker(A - I) = \text{Span}\left\{\begin{bmatrix} 1 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right\} \Rightarrow \lambda_2 = 1 \text{ geom. mult. } 2 \end{array}\right\} \Rightarrow A \text{ - diagonalizable}$$

& $\left\{\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right\}$ is an eigenbasis

$\boxed{20}$ $\lambda_1 = 1$ alg. mult 2

$\lambda_2 = 2$ alg. mult 1

$$E_1 = \ker(A - I) = \ker\left\{\begin{bmatrix} 0 & a & b \\ 0 & 0 & c \\ 0 & 0 & 1 \end{bmatrix}\right\} \Rightarrow \text{which has } \begin{cases} \dim = 2, \text{ if } a = 0 \Rightarrow \lambda_1 \text{ has geom. mult } 2 \\ \dim = 1, \text{ if } a \neq 0 \Rightarrow \lambda_1 \text{ -''- } 1 \end{cases}$$

$$E_2 = \ker(A - 2I) = \ker\left\{\begin{bmatrix} -1 & a & b \\ 0 & -1 & c \\ 0 & 0 & 0 \end{bmatrix}\right\}, \text{ which has dim 1 always} \Rightarrow \lambda_2 \text{ - has geom. mult. } 1$$

$\Rightarrow$ A - diagonalizable $\Leftrightarrow a = 0$.

(it has eigenbasis)

$\boxed{24}$ $A \cdot \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} a & 2-2a \\ c & 1-2c \end{bmatrix}$, $c_A(x) = (x-1)^2 = x^2 - 2x + 1 \Rightarrow \begin{cases} \text{tr}(A) = 2 \\ \det A = 1 \end{cases} \Rightarrow$

(since it has only 1 eigenvalue)

$\Rightarrow a = 1 + 2c \Rightarrow A = \begin{bmatrix} 1+2c & -4c \\ c & 1-2c \end{bmatrix}$. Now $\ker(A - I) = \ker\left\{\begin{bmatrix} 2c & -4c \\ c & -2c \end{bmatrix}\right\} = \text{Span}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$,

which happens only when $c \neq 0$. $\Rightarrow$ $\boxed{A = \begin{bmatrix} 1+2c & -4c \\ c & 1-2c \end{bmatrix}, c \neq 0.}$