HW 10

6.1 **18** $\det\begin{bmatrix}0&1&k\\3&2k&5\\9&7&5\end{bmatrix} = 30+21k-18k^2 = -3(k-2)(6k+5)$

$\Rightarrow k\neq 2$ or $k\neq -\frac{5}{6}$

29 $\det(A-\lambda I_3) = (2-\lambda)(3-\lambda)(5-\lambda)=0$ if $\lambda\in\{2,3,5\}$

50 • $|\det A| = \|v_1\|\cdot\|v_2^{\perp}\|\cdot\|v_3^{\perp}\| \leq \|v_1\|\cdot\|v_2\|\cdot\|v_3\| = 1$

$\Rightarrow \det A\in[-1,1]$

Equality is attained if $\|v_2\|=\|v_2^{\perp}\|$ & $\|v_3\|=\|v_3^{\perp}\|$, i.e. if A is orthogonal.

OR:

$\det A = \vec{u}\cdot(\vec{v}\times\vec{w}) = \|\vec{u}\|\cdot\|\vec{v}\times\vec{w}\|\cos\theta = \underset{1}{\|v\|}\cdot\underset{1}{\|w\|}\cdot\sin\varphi\cos\theta = \sin\varphi\cos\theta\in[-1,1]$

$\theta = \measuredangle(\vec{u},\vec{v}\times\vec{w})$

$\varphi = \measuredangle(\vec{v},\vec{w})$

6.2 **8** $\det A = 2$

16 $\det\begin{bmatrix}6\vec{v}_1+2\vec{v}_4\\ \vec{v}_2\\ \vec{v}_3\\ 3\vec{v}_1+\vec{v}_4\end{bmatrix} \overset{R_1-2R_4}{=} \det\begin{bmatrix}\vec{0}\\ \vec{v}_2\\ \vec{v}_3\\ 3\vec{v}_1+\vec{v}_4\end{bmatrix} = 0$

28 A basis of the plane is $\left\{\begin{bmatrix}-2\\1\\0\end{bmatrix},\begin{bmatrix}-3\\0\\1\end{bmatrix}\right\}$

$T\left(\begin{bmatrix}-2\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}\times\begin{bmatrix}-2\\1\\0\end{bmatrix} = \begin{bmatrix}-3\\-6\\5\end{bmatrix} = -6\begin{bmatrix}-2\\1\\0\end{bmatrix}+5\begin{bmatrix}-3\\0\\1\end{bmatrix}$

$T\left(\begin{bmatrix}-3\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\2\\3\end{bmatrix}\times\begin{bmatrix}-3\\0\\1\end{bmatrix} = \begin{bmatrix}2\\-10\\6\end{bmatrix} = -10\begin{bmatrix}-2\\1\\0\end{bmatrix}+6\begin{bmatrix}-3\\0\\1\end{bmatrix}$

$\Rightarrow A = \begin{bmatrix}-6&-10\\5&6\end{bmatrix}$

$\det A = 14$

46 a) $\det\begin{bmatrix}a&3&d\\b&3&e\\c&3&f\end{bmatrix} = 3\det\begin{bmatrix}a&1&d\\b&1&e\\c&1&f\end{bmatrix} = 3\cdot 7 = 21$

b) $\det\begin{bmatrix}a&3&d\\b&5&e\\c&7&f\end{bmatrix} = \det\begin{bmatrix}a & 2\begin{bmatrix}1\\2\\3\end{bmatrix}+\begin{bmatrix}1\\1\\1\end{bmatrix} & d\\ b & & e\\ c & & f\end{bmatrix} = 2\det\begin{bmatrix}a&1&d\\b&2&e\\c&3&f\end{bmatrix} + \det\begin{bmatrix}a&1&d\\b&1&e\\c&1&f\end{bmatrix} = 2\cdot 11+7 = 29$

50 $\det M_n = \det\begin{bmatrix}1&1&1&\dots&1\\1&2&2&\dots&2\\1&2&3&\dots&3\\\vdots&\vdots&\vdots&&\vdots\\1&2&3&\dots&n\end{bmatrix} \underset{j=\overline{2,n}}{\overset{R_j-R_1}{=}} \det\begin{bmatrix}1&1&1&\dots&1\\0&1&1&\dots&1\\0&1&2&\dots&2\\\vdots&&&&\\0&1&2&\dots&(n-1)\end{bmatrix} = 1\cdot(-1)^{1+1}\det\begin{bmatrix}1&1&\dots&1\\1&2&\dots&2\\\vdots&\vdots&&\vdots\\1&2&\dots&(n-1)\end{bmatrix} = \det M_{n-1},\ \forall n\in\mathbb{N}$

Hence $\det M_n = \det M_{n-1} = \det M_{n-2} = \dots = \det M_2 = \det M_1 = 1$

6.3 **10** $|\det A| = \|v_1\|\cdot\|v_2^{\perp}\|\cdot\ldots\cdot\|v_n^{\perp}\| \leq \|v_1\|\cdot\|v_2\|\cdot\ldots\cdot\|v_n\|$. Equality holds if $\|\vec{v}_i\| = \|\vec{v}_i^{\perp}\|$, i.e. $\{v_1,\dots,v_n\}$ – orthogonal (on each other)

30 $\operatorname{adj}A = \begin{bmatrix}18&0&0\\-12&6&0\\-2&-5&3\end{bmatrix}$
