Math 2210

Spring 2009

Prelim 2

Name:

Please indicate your lecture section by circling the time when class meets.

9:05-9:55	10:10-11:00	11:15-12:05

Directions:

Complete all seven questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use a calculator.

You may not use any notes.

You have 90 minutes.

Problem 1: _____ / 10

Problem 2: _____ / 5

Problem 3: _____ / 5

Problem 4: _____ / 10

Problem 5: _____ / 10

Problem 6: _____ / 5

Problem 7: _____ / 5

Total: _____ / 50

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation $T(x) = Ax$ given by the matrix

$$A = \begin{bmatrix} -3 & -4 \\ 2 & 3 \end{bmatrix}.$$

(a) Give the matrix B of the transformation T in the basis $\mathcal{B} = \left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right)$.

(b) Compute A^{1776}. It may help to use your result from part (a).

2. Let V be the vector space (linear space) of all 2×2 matrices, and consider the linear transformation $T : V \to \mathbb{R}$ given by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + d.$$

Using the inner product

$$\left\langle \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \right\rangle = aa' + bb' + cc' + dd',$$

compute an orthonormal basis of the kernel of T.

3. Does there exist an orthogonal transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that:

$$T\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \text{ and } T\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}?$$

If yes, give a basis for $\mathbb{R}^3$ (of your choice) and give the matrix of such a transformation T with respect to that basis. If not, explain why.

4. (a) Show that the following system has no exact solution:

$$\begin{cases} x + y &= 29 \\ x - y &= -9 \\ 2x &= 22 \end{cases}$$

(b) Give the best approximation to a solution in the least squares sense.

5. Let n be a positive integer, and let P_n be the vector space (linear space) of polynomials of degree less than or equal to n.

   (a) Let $T : P_n \to P_n$ be the linear transformation defined by

$$T(f(x)) = x\frac{d}{dx}(f(x)).$$

   Show that T is not an isomorphism for any n.

   (b) Let $S : P_3 \to P_3$ be the linear transformation defined by

$$S(f(x)) = \frac{d}{dx}(xf(x)).$$

   Determine whether or not S is an isomorphism.

6. Let V be the vector space (linear space) of all symmetric 2×2 matrices. This is a finite dimensional vector space, and hence is isomorphic to $\mathbb{R}^n$ for some n. Determine n and give an isomorphism from V to $\mathbb{R}^n$.

7. Let A be a matrix with linearly independent columns and let $A = QR$ be a QR factorization of A. Is $Q^T Q$ necessarily the identity? What about QQ^T?