

Math 2210
Spring 2009
Prelim 1

Name:

Directions:

Complete all seven questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use a calculator.

You may not use any notes.

You have 90 minutes.

Problem 1: _____ / 10

Problem 2: _____ / 10

Problem 3: _____ / 5

Problem 4: _____ / 5

Problem 5: _____ / 10

Problem 6: _____ / 10

Problem 7: _____ / 10

Total: _____ / 60

1. Give bases for the image and kernel of the following matrix

$$\begin{bmatrix} 2 & 4 & -4 & 3 \\ -1 & -2 & 2 & -1 \\ 1 & 3 & -4 & 3 \end{bmatrix}$$

- Let A and B be $n \times n$ matrices. Anthony claims that $\ker A$ is always a subset of $\ker AB$. Brian claims that $\ker B$ is always a subset of $\ker AB$. Exactly one of them is right. Which one? Be sure to fully justify your answer.

3. Let A be a 9×6 matrix. Suppose that $2 \dim(\ker A) = \dim(\operatorname{im} A)$. Find the rank of A .

4. Let W be a subspace of \mathbb{R}^7 and let $\vec{u}_1, \vec{u}_2, \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4 \in W$. Suppose that $\{\vec{u}_1, \vec{u}_2\}$ is a linearly independent set and $\{\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4\}$ spans W . If neither of these sets is a basis, find $\dim W$.

5. Consider the following system of equations, where a is a parameter in \mathbb{R} .

$$\begin{cases} x & & -2z & = & -4 \\ -x & +y & +(a+4)z & = & 10 \\ x & -ay & & -5z & = & -10 \end{cases}$$

- (a) For which values of a does the system have a unique solution?
- (b) For which values of a does the system have no solution?
- (c) For which values of a does the system have infinitely many solutions?

6. Consider a linear system of n equations and m variables which has rank r . Answer the three following questions for (n, m, r) taking values: $(7, 5, 4)$, $(7, 5, 5)$, $(5, 7, 4)$, $(5, 7, 5)$.

(a) Must the system have at least one solution ?

(b) Must the system have at most one solution ?

(c) Can the system have a unique solution ?

Answer by Yes or No in a table such as:

(n, m, r)	(1)	(2)	(3)
$(7, 5, 4)$			
$(7, 5, 5)$			
$(5, 7, 4)$			
$(5, 7, 5)$			

7. Compute the inverse of the matrix A . Note that your answer should involve the parameters a, b and c :

$$A = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$