Math 221, Preliminary Exam 1

September 27, 2005

Name:

Lecture time:

Instructor:

Problem 1.

Solve the following system of linear equations using Gauss-Jordan elimination.

$$\begin{array}{ccccccc} x & + & 2y & + & 3z & = & 8 \\ x & + & 4y & + & 3z & = & 10 \\ x & + & 2y & + & 4z & = & 9 \end{array}$$

Problem 2.

(a) Find the matrix A of the following linear transformation.

$$\begin{aligned} y_1 &= x_1 + 2x_2 \\ y_2 &= x_1 + x_2 \\ y_3 &= 4x_3 \end{aligned}$$

(b) Find A^{-1}.

(c) Find A^2.

Problem 3.

(a) Define the following two terms:

(i) rank of a matrix, and

(ii) kernel of a matrix.

Let $B = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 4 \end{bmatrix}$.

(b) Find the rank of the matrix B.

(c) Find the kernel of the matrix B.

Problem 4.

Let $v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} 2 \\ 11 \end{bmatrix}$.

(a) Write v_3 as a linear combination of v_1 and v_2.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^4$ be a linear transformation such that

$$T(v_1) = \begin{bmatrix} 0 \\ 2 \\ 3 \\ -2 \end{bmatrix} \text{ and } T(v_2) = \begin{bmatrix} 4 \\ -9 \\ 0 \\ -1 \end{bmatrix}.$$

Find $T(v_3)$.

Problem 5.

Prove that $T : \mathbb{R}^2 \to \mathbb{R}^3$, defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 7x_1x_2 \\ -3x_2 \\ 4x_1 + x_2 \end{bmatrix},$$

is not linear.

Problem 6.

Consider the following augemented matrix of a linear system.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & a & b \end{array}\right]$$

For what a and b does this system have

(a) no solution,

(b) exactly one solution,

(c) infinitely many solutions?

Problem 7.

Find the matrix of the linear transformation that sends the picture on the left to the picture on the right.