# HW 12

**7.4** **26** The eigenvalues are 1, 2, 1 so $A$ is diagonalizable if & only if $E_1$ has dim 2.

Now $E_1 = \ker(A - I_3) = \ker\begin{bmatrix} 0 & a & b \\ 0 & 1 & c \\ 0 & 0 & 0\end{bmatrix} = \ker\begin{bmatrix} 0 & 1 & c \\ 0 & 0 & b-ac \\ 0 & 0 & 0\end{bmatrix}$ is 2-dim $\Leftrightarrow b = ac$.

$$\underline{A\text{ - diagonalizable} \Leftrightarrow b = ac}$$

**42** · If $A$ - symmetric $\Rightarrow T(A) = A - A^T = A - A = 0 \Rightarrow$ it's an eigenvector with eigenvalue 0.

If $A$ - skew-symmetric $\Rightarrow T(A) = A - A^T = A - (-A) = 2A \Rightarrow$ it's an eigenvector with eigenvalue 2.

A basis for symmetric matrices is $\left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&1\\1&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$, while one for skew-symmetric matrices is $\left\{\begin{bmatrix}0&1\\-1&0\end{bmatrix}\right\}$. There are 4 (basis) eigenvectors, so $\underline{T \text{ is diagonalizable}}$.

**50** W.r.t. $\{1, x, x^2\}$, $T$ has the matrix $B = \begin{bmatrix} 1 & -3 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1\end{bmatrix}$. $c_B(x) = (x-1)^3$, so $\lambda = 1$, with alg. mult. 3

$\ker(B - I_3) = \operatorname{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\} = \operatorname{Span}\{1\} \Rightarrow \underline{T \text{ - not diagonalizable}}$

**56** Using the hint:
$$\begin{bmatrix} AB & 0 \\ B & 0\end{bmatrix}\begin{bmatrix} I & A \\ 0 & I\end{bmatrix} = \begin{bmatrix} I & A \\ 0 & I\end{bmatrix}\begin{bmatrix} 0 & 0 \\ B & BA\end{bmatrix} \quad (1)$$

But
$$\begin{bmatrix} -xI & 0 \\ 0 & -xI\end{bmatrix}\cdot\begin{bmatrix} I & A \\ 0 & I\end{bmatrix} = \begin{bmatrix} I & A \\ 0 & I\end{bmatrix}\begin{bmatrix} -xI & 0 \\ 0 & -xI\end{bmatrix} \quad (2)$$

Add (1) & (2) & factor $\Rightarrow \begin{bmatrix} AB - xI & 0 \\ B & -xI\end{bmatrix}\begin{bmatrix} I & A \\ 0 & I\end{bmatrix} = \begin{bmatrix} I & A \\ 0 & I\end{bmatrix}\begin{bmatrix} -xI & 0 \\ B & BA - xI\end{bmatrix} \rightsquigarrow$ Take determinants $\Rightarrow$

$$\det(AB - xI)\cdot\cancel{\det(-xI)}\cdot\det^2(I) = \det^2(I)\cdot\cancel{\det(-xI)}\cdot\det(BA - xI)$$

Hence $\det(AB - xI) = \det(BA - xI)$, so $AB$ & $BA$ have the $\underline{\text{same characteristic polynomials}}$.

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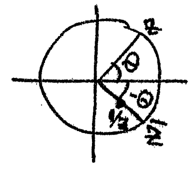
**7.5** **6** $z = r(\cos\theta + i\sin(\theta)) \Rightarrow \dfrac{1}{z} = \dfrac{1}{r}\left(\cos(-\theta) + i(\sin(-\theta))\right)$




$\bar{z} = r(\cos\theta - i\sin(\theta)) = r(\cos(-\theta) + i\sin(-\theta))$

**24** $c_A(x) = -x^3 + 3x^2 - 7x + 5 \Rightarrow \underline{\lambda_1 = 1,\ \lambda_2 = 1 + 2i,\ \lambda_3 = 1 - 2i}$

**28** Let $\left.\begin{matrix}\lambda_1 = a + ib \\ \lambda_2 = a - ib\end{matrix}\right\} \Rightarrow \begin{matrix} \operatorname{tr}(A) = \lambda_1 + \lambda_2 + 2 = 2 + 2a = 8 \Rightarrow a = 3 \\ \det A = 2\cdot\lambda_1\lambda_2 = 2(a^2 + b^2) = 50 \Rightarrow b = 4\end{matrix} \Rightarrow \underline{\lambda_{1,2} = 3 \pm 4i}$

**50** $\lambda_1 = 0$ (alg. mult. 2), $\lambda_2 = 1$. $\ker(A) = \operatorname{Span}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$ - 2-dim. $\Rightarrow$ It's $\underline{\text{diagonalizable for any value of } a}$.

$\boxed{7.6}$ $\boxed{8}$ $\lambda_1 = 0.9$, $\lambda_2 = 0.8$ $\Rightarrow$ $\vec{0}$ - stable equilibrium

$\boxed{36}$ „$\Rightarrow$" If zero is stable $\Rightarrow$ $\lim_{t\to\infty}$ ($i$th column of $A^t$) $= \lim_{t\to\infty} (A^t e_i) = \vec{0}$ $\Rightarrow$ All entries of $A^t$ approach 0.

„$\Leftarrow$" If $\lim_{t\to\infty} A^t = 0$ $\Rightarrow$ $\lim_{t\to\infty} (A^t x_0) = (\lim_{t\to\infty} A^t)\vec{x}_0 = \vec{0}$ for all $\vec{x}_0$ $\Rightarrow$ the zero state is stable.