HW 1

1.1 **8**

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 4 & 5 & 6 & 0 \\ 7 & 8 & 10 & 0 \end{array}\right] \xrightarrow[R_3 - 7R_1]{R_2 - 4R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -3 & -6 & 0 \\ 0 & -6 & -11 & 0 \end{array}\right] \xrightarrow[R_3 - 2R_2]{-R_2/3} \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow$$

$$\xrightarrow[R_1 - 2R_2 + R_1]{R_2 - 2R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

16 The system reduces to:

$$\begin{cases} x + 5z = 0 \\ y - z = 0 \\ 0 = 0 \end{cases} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = t \begin{pmatrix} -5 \\ 1 \\ 1 \end{pmatrix}, \ t \in \mathbb{R}$$

(a line)

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$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 1 & 3 & 8 & b \\ 1 & 2 & 2 & c \end{array}\right] \xrightarrow[R_3 - R_1]{R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 2 & 3 & a \\ 0 & 1 & 5 & b-a \\ 0 & 0 & -1 & c-a \end{array}\right] \xrightarrow[\substack{R_2 + 5R_3 \\ R_3 \cdot (-1)}]{R_1 - 2R_2 - 7R_3}$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 10a - 2b - 7c \\ 0 & 1 & 0 & b + 5c - 6a \\ 0 & 0 & 1 & a - c \end{array}\right] \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10a - 2b - 7c \\ -6a + b + 5c \\ a - c \end{pmatrix}$$

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$$\begin{cases} a + b + c = p \\ a + 2b + 4c = q \\ a + 3b + 9c = r \end{cases} \Rightarrow \begin{cases} a = 3p - 3q + r \\ b = -\frac{5}{2}p + 4q - \frac{3}{2}r \\ c = \frac{1}{2}p - q + \frac{1}{2}r \end{cases} \Rightarrow \text{Unique polynomial:}$$

$$f(t) = (3p - 3q + r) + \left(-\tfrac{5}{2}p + 4q - \tfrac{3}{2}r\right)t + \left(\tfrac{1}{2}p - q + \tfrac{1}{2}r\right)t^2$$

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$$\begin{cases} b + \frac{m}{2} = c \\ \frac{1}{2}b + m = 2c \end{cases} \Rightarrow \begin{cases} b = 0 \\ m = 2c \end{cases} \Rightarrow \text{Boris had no money.}$$

1.2 **6**

$$\begin{cases} x_1 = 3 + 7x_2 - x_5 \\ x_3 = 2 + 2x_5 \\ x_4 = 1 - x_5 \end{cases}, \quad \text{let } x_2 = t,\ x_5 = r \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 7 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 0 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

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$$\begin{cases} x_2 = x_5 \\ x_4 = -2x_5 \end{cases} \quad \text{Let } x_1 = r,\ x_3 = s,\ x_5 = t \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = r \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

12 After row-reducing:

$$\begin{cases} x_1 = -\frac{7}{2}x_5 - x_6 \\ x_2 = -x_5 \\ x_3 = \frac{5}{3}x_6 \\ x_4 = -3x_5 - x_6 \end{cases} \quad \text{Let } x_5 = r,\ x_6 = t \Rightarrow$$

$$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_6 \end{bmatrix} = r \begin{bmatrix} -7/2 \\ -1 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ 0 \\ 5/3 \\ -1 \\ 0 \\ 1 \end{bmatrix}$$

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$$\begin{cases} a = 1 \\ a + b + c + d = 0 \\ a - b + c - d = 0 \\ a + 2b + 4c + 8d = -15 \end{cases} \Rightarrow \begin{cases} a = 1 \\ b = 2 \\ c = -1 \\ d = -2 \end{cases} \Rightarrow f(t) = 1 + 2t - t^2 - 2t^3$$

$\boxed{46}$ a) $\left[\begin{array}{ccc|c} 0 & 1 & 2k & 0 \\ 1 & 2 & 6 & 2 \\ k & 0 & 2 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ k & 0 & 2 & 1 \end{array}\right] \xrightarrow{R_3 - kR_1} \left[\begin{array}{ccc|c} 1 & 2 & 6 & 2 \\ 0 & 1 & 2k & 0 \\ 0 & -2k & 2-6k & 1-2k \end{array}\right] \rightarrow$

$\xrightarrow[R_1 - 2R_2]{R_3 + (2k)\cdot R_2} \left[\begin{array}{ccc|c} 1 & 0 & 6-4k & 2 \\ 0 & 1 & 2k & 0 \\ 0 & 0 & 2-6k+4k^2 & 1-2k \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 6-4k & 2 \\ 0 & 1 & 2k & 0 \\ 0 & 0 & 2(2k-1)(k-1) & -(2k-1) \end{array}\right]$

- Unique sol: $(2k-1)(k-1) \neq 0 \Rightarrow \boxed{k \neq \frac{1}{2} \ \& \ k \neq 1}$

b) No sol.: $(2k-1)(k-1) = 0$ but $(2k-1) \neq 0 \Rightarrow \boxed{k = 1}$

c) Infinite sol.: $(2k-1)(k-1) = 0$ and $2k-1 = 0 \Rightarrow \boxed{k = \frac{1}{2}}$