MATH 221, FALL 2007, 2ND PRELIM

OCTOBER 25, 2007.

No books, notes or calculators. Show all work. Write clearly.

There are 100 points on the test plus 5 points extra credit.

1. (20 points) Let P_2 denote the linear space of all polynomials in t of degree at most two. Let T be the linear transformation from P_2 to itself given by

$$T(f) = f' + f''.$$

a) Find the matrix of T in the standard basis $1, t, t^2$ for P_2.

b) What is the determinant of T?

2. (15 points) Is the linear space P_9 of polynomials of degree at most 9 isomorphic to the linear space $\mathbb{R}^{3\times 3}$ of 3 by 3 matrices? Justify your answer.

3. (10 points) Which, if any, of the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

is orthogonal? Justify your answer.

4. (20 points) Let V be the plane in $\mathbb{R}^3$ spanned by the vectors

$$v = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad w = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

a) Find the matrix of the orthogonal projection of $\mathbb{R}^3$ onto V.

b) What vector in V best approximates the vector $x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ in the least squares sense?

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5. (15 points) Let $C[-\pi,\pi]$ be the linear space of all continuous functions on $[-\pi,\pi]$ with the inner product

$$\langle f, g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)g(t)\,dt.$$

Find the distance between functions $\sin t$ and $\cos t$ in this space.

6. (20 points) Let A be the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}.$$

a) Find the QR factorization of A.

b) What is the area of the parallelogram defined by the columns of A?

7. (extra credit, 5 points) Evaluate the determinant

$$\det \begin{bmatrix} 1 & 1 & 1 & 1 & \dots & 1 & 1 \\ 1 & 2 & 2 & 2 & \dots & 2 & 2 \\ 1 & 2 & 3 & 3 & \dots & 3 & 3 \\ 1 & 2 & 3 & 4 & \dots & 4 & 4 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \\ 1 & 2 & 3 & 4 & \dots & n-1 & n-1 \\ 1 & 2 & 3 & 4 & \dots & n-1 & n \end{bmatrix}.$$