Math 2210

Spring 2009

Prelim 1

Name:

Directions:

Complete all seven questions.

Show your work. A correct answer without any scratch work or justification may not receive much credit.

You may not use a calculator.

You may not use any notes.

You have 90 minutes.

Problem 1:	_____	/ 10
Problem 2:	_____	/ 10
Problem 3:	_____	/ 5
Problem 4:	_____	/ 5
Problem 5:	_____	/ 10
Problem 6:	_____	/ 10
Problem 7:	_____	/ 10
Total:	_____	/ 60

1. Give bases for the image and kernel of the following matrix.

$$\begin{bmatrix} 2 & 3 & 3 & -2 \\ 1 & 2 & 1 & 2 \\ -1 & 3 & -6 & 1 \end{bmatrix}$$

2. Let A and B be $n \times n$ matrices. Amy claims that ker A is always a subset of ker BA. Barbara claims that ker B is always a subset of ker BA. Exactly one of them is right. Which one? Be sure to fully justify your answer.

3. Let A be a 9×15 matrix. Suppose that $\dim(\ker A) = 2 \dim(\operatorname{im} A)$. Find the rank of A.

4. Let W be a subspace of $\mathbb{R}^6$ and let $\vec{u_1}, \vec{u_2}, \vec{u_3}, \vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}, \vec{v_5} \in W$. Suppose that $\{\vec{u_1}, \vec{u_2}, \vec{u_3}\}$ is a linearly independent set and $\{\vec{v_1}, \vec{v_2}, \vec{v_3}, \vec{v_4}, \vec{v_5}\}$ spans W. If neither of these sets is a basis, find $\dim W$.

5. Consider the following system of equations, where a is a parameter in $\mathbb{R}$.

$$\begin{cases} x & & -3z & = & 3 \\ -x & +y & +(a+2)z & = & 1 \\ x & -ay & -5z & = & -5 \end{cases}$$

(a) For which values of a does the system have a unique solution?

(b) For which values of a does the system have no solution?

(c) For which values of a does the system have infinitely many solutions?

6. Consider a linear system of n equations and m variables which has rank r. Answer the three following questions for (n, m, r) taking values: $(7, 6, 4)$, $(6, 7, 4)$, $(7, 6, 6)$, $(6, 7, 6)$.

   (a) Must the system have at least one solution ?

   (b) Must the system have at most one solution ?

   (c) Can the system have a unique solution ?

   Answer by Yes or No in a table such as:

(n, m, r)	(1)	(2)	(3)
$(7, 6, 4)$			
$(6, 7, 4)$			
$(7, 6, 6)$			
$(6, 7, 6)$			

7. Compute the inverse of the matrix A. Note that your answer should involve the parameters a, b and c:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$