Math 2130 Workshop: Circulation Around a Region

In this workshop, compute circulation counterclockwise unless otherwise specified.

1) Suppose $C$ is some loop, $\vec{F}$ is a vector field and $P$ and $Q$ are points on the loop. Argue that:

- If the vector line integral of $\vec{F}$ one way around the loop from $P$ to $Q$ is equal to the vector line integral of $\vec{F}$ the other way around the loop from $P$ to $Q$ then the circulation of $\vec{F}$ around the loop is 0.
- If the circulation of $\vec{F}$ around the loop is 0 then the vector line integral of $\vec{F}$ one way around the loop from $P$ to $Q$ is equal to the vector line integral of $\vec{F}$ the other way around the loop from $P$ to $Q$.

2) Show that the circulation of a vector field $\vec{F}$ around the boundary of the combined region $A \cup B$ below is the circulation of $\vec{F}$ around the boundary of $A$ plus the circulation of $\vec{F}$ around the boundary of $B$. 

![Diagram of regions A and B with a combined boundary]
3) Which way should we orient the inner boundary of the region shown below such that the circulation of a vector field \( \vec{F} \) around the boundary of the combined region \( A \cup B \) is the circulation of \( \vec{F} \) around the boundary of \( A \) plus the circulation of \( \vec{F} \) around the boundary of \( B \)?

![Diagram](image)

4) Jigsaw puzzles are (usually) made of a grid of square-like pieces with each edge either having an inward or outward facing connector ("innies" and "outies"). An example of some interconnected jigsaw puzzle pieces is shown below. Associate to each puzzle piece the number of outies minus the number of innies. The values of some pieces are:

\[
\begin{array}{cccc}
4 & 2 & 0 & -2 \\
-2 & -4 & & \\
\end{array}
\]

Argue that if you add up this value for all the puzzle pieces put together you get the number of outies minus the number of innies on the perimeter of the region.

![Jigsaw Puzzle](image)