Math 2130 Workshop: Sensitivity Analysis

For systems with a dependent variable which depends on a number of independent variables, partial derivatives allow us to study how much small changes in each of the independent variables will get magnified in their effect on the dependent variable.

1) Write out the formula for the volume of a cylinder V as a function of its diameter d and height h. Compute $\frac{\partial V}{\partial d}$ and $\frac{\partial V}{\partial h}$.

2) Compute a linear approximation to V at $d = 6$ cm and $h = 10$ cm.

3) Using your linear approximation, decide which would have a bigger effect on the volume: a .1 cm increase in the diameter, or a .1 cm increase in the height.

4) Based on your solution to problem **1**, does your answer to problem **3** change for different values of d and h?

5) You are conducting a physics experiment to measure the gravitational acceleration constant g. You drop a weight from a height of h meters off the ground and it takes t seconds for it to hit the ground. You compute $g = \frac{2h}{t^2}$. Compute $\frac{\partial g}{\partial h}$ and $\frac{\partial g}{\partial t}$.

6) You know g is roughly $1000\frac{\text{cm}}{\text{s}^2}$, and want to design an experiment ahead of time to get the best measurement of g with imprecise instruments. Your first plan is to drop the weight from roughly 500 centimeters above the ground so it takes roughly 1 second to hit the ground. Based on your answer to question **5**, which would have a bigger effect on your calculation of g: a .1 second error or a .1 cm error?

7) Calculate your partials again for a height of roughly 125 cm and time of roughly $\frac{1}{2}$ second. Which has a bigger effect on your calculaton: a 10 percent error in your measurement of time for this experiment (.05 seconds), or a 10 percent error in your previous experiment (as in problem **6**).