Math 2130 Prelim 2 (PRACTICE) (Spring 2017)

(1)	______/3
(2)	______/3
(3)	______/3
(4)	______/3
(5)	______/3
(6)	______/3
(7)	______/3
(8)	______/3
Total	______/24

Before the exam:

- **Do not write anything on this page.**
- Do not open the exam.
- Turn off your cell phone.
- Make sure your books, notes, and electronics are not visible during the exam.
- Do not wear headphones during the exam.

When you open your exam:

- Make sure your exam has all its pages. There are 6 pages, including the last, and 8 problems. **The real exam will likely be shorter.**
- If you believe there is a printing error, let me know right away.
- Write your name on the last page, and put a check in the box corresponding to your section.

During the exam:

- Do not talk or ask questions. If you are unsure what a question is asking, demonstrate your understanding as best you can.
- Be respectful of your fellow classmates.
- You may use the bathroom during the exam, but please ask first so I can keep track of who is out of the room at any one time.
- If you finish your exam before 2:00, you may leave early: hand your exam in at the front of the room, and do not discuss the exam directly outside the classroom. If you finish after 2:00, please remain quiet and seated until 2:15.

Notes on grading:

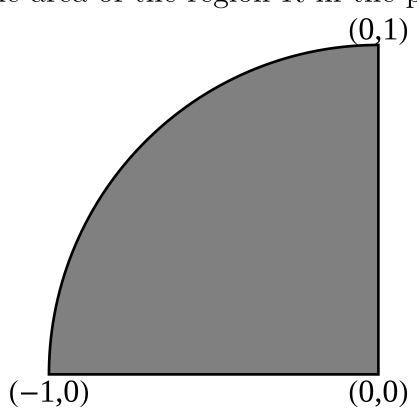
- Draw a $\boxed{\text{box}}$ around your final solution to the problem.
- Show your work. Demonstrate that you know how to get the correct answer, not just make a lucky guess.
- Clearly cross out any work that is incorrect.
- Problems will be graded out of 3.
- If you run out of room, continue your work on the back of the previous page. Make a note that you've done this, and make it clear where your work continues.

(1) Set up (but do not evaluate) iterated integrals using spherical coordinates to compute the average y-coordinate of the region R given by half sphere $x^2+y^2+z^2 \leq 1$ and $z \leq 0$.

$$\frac{\int_{\theta=0}^{\theta=2\pi} \int_{\phi=\frac{\pi}{2}}^{\phi=\pi} \int_{\rho=0}^{\rho=1} (\rho \sin\phi \sin\theta)(\rho^2 \sin\phi) d\rho d\phi d\theta}{\int_{\theta=0}^{\theta=2\pi} \int_{\phi=\frac{\pi}{2}}^{\phi=\pi} \int_{\rho=0}^{\rho=1} \rho^2 \sin\phi d\rho d\phi d\theta}$$

It would also be acceptable to just know the volume of a half-sphere and use that as the denominator.

(2) Set up (but do not evaluate) iterated integrals using polar coordinates to compute the area of the region R in the picture:



$$\int_{\theta=\frac{\pi}{2}}^{\theta=\pi} \int_{r=0}^{r=1} r dr d\theta$$

(3) Switch the order of integration of $\int_{x=0}^{x=1} \int_{y=x}^{y=1} xy^2 \ dy \ dx$. Do not evaluate your result.

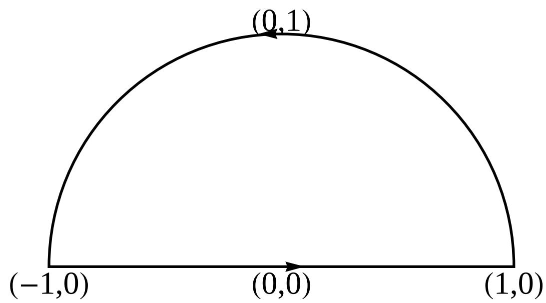
$$\int_{y=0}^{y=1} \int_{x=0}^{x=y} xy^2 dx dy$$

(4) Set up (but do not evaluate) an integral to find the arc length of the half-circle $x^2 + y^2 = 1$ and $y \geq 0$. Leave any vector operations unevaluated.

Using $\vec{r}(t) = (\cos t, \sin t)$ (for $0 \leq t \leq \pi$) we have that $\vec{r}\,'(t) = (-\sin t, \cos t)$:

$$\int_{t=0}^{t=\pi} ||(-\sin t, \cos t)|| dt$$

(5) Set up (but do not evaluate) integrals to find the circulation (counterclockwise) of the vector field $\vec{F} = (-y^2, x^5)$ over the curve C in the picture below. Leave any vector operations unevaluated.



For the upper curve, we use $\vec{r}(t) = (\cos t, \sin t)$ (for $0 \leq t \leq \pi$) ($\vec{r}\,'(t) = (-\sin t, \cos t)$). For the lower curve, we use $\vec{r}(t) = (t, 0)$ for $-1 \leq t \leq 1$ ($\vec{r}\,'(t) = (1, 0)$). Our integrals are:

$$\int_{t=0}^{t=2\pi} (-\sin^2 t, \cos^5 t) \cdot (-\sin t, \cos t) dt + \int_{t=-1}^{t=1} (0, t^5) \cdot (1, 0) dt$$

(6) Find the critical point of $f(x, y) = -3 - 6x + 4y + 2xy - y^2$. What does the second derivative test tell you about the critical point?

$f_x = -6 + 2y$ and $f_y = 4 + 2x - 2y$. These are never undefined. Solving for when they are 0, we get $y = 3$ and $x = 1$. $f_{xx} = 0$, $f_{yy} = -2$, $f_{xy} = 2$. So $D = (0)(-2) - (2)^2 = -4$. (Note: in general D might depend on x and y, but in this case it does not.) So f has a saddle point at $(3, 1)$.

(7) Use Lagrange multipliers to set up (but do not solve) algebraic equations you could solve to find points of interest of $f(x, y) = x^2+y^2$ given the constraint $2x^2+y^2 = 1$. Hint: you need as many equations as unknowns.

Letting $g(x, y) = 2x^2 + y^2$, from $\nabla f = \lambda \nabla g$ we get the two equations:

$$2x = \lambda 4x$$
$$2y = \lambda 2y$$

We also need to add in our original equation to get three equations and three unknowns:

$$2x^2 + y^2 = 1$$

(8) Evaluate $\int_{y=2}^{y=3} \int_{x=0}^{x=1} x^y \, dx \, dy$.

$$\begin{aligned}
&\int_{y=2}^{y=3} \int_{x=0}^{x=1} x^y \, dx \, dy \\
&= \int_{y=2}^{y=3} \left[\frac{x^{y+1}}{y+1} \right]_{x=0}^{x=1} dy \\
&= \int_{y=2}^{y=3} \frac{1}{y+1} \, dy \\
&= [\ln |y+1|]_{y=2}^{y=3} \\
&= \ln 4 - \ln 3
\end{aligned}$$

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Name: ______________________________ Netid: ________________

Section (check which one applies):

☐ Discussion 1 (9:05am-9:55am)

☐ Discussion 2 (10:10am-11:00am)