# Math 2130 Prelim 1 (Spring 2017)

Before the exam:

- Do not write anything on this page.
- Do not open the exam.
- Turn off your cell phone.
- Make sure your books, notes, and electronics are not visible during the exam.
- Do not wear headphones during the exam.

When you open your exam:

- Make sure your exam has all its pages. There are 6 pages, including the last, and 8 problems.
- If you believe there is a printing error, let me know right away.
- Write your name on the last page, and put a check in the box corresponding to your section.

During the exam:

- Do not talk or ask questions. If you are unsure what a question is asking, demonstrate your understanding as best you can.
- Be respectful of your fellow classmates.
- You may use the bathroom during the exam, but please ask first so I can keep track of who is out of the room at any one time.
- If you finish your exam before 2:00, you may leave early: hand your exam in at the front of the room, and do not discuss the exam directly outside the classroom. If you finish after 2:00, please remain quiet and seated until 2:15.

Notes on grading:

- Draw a $\boxed{\text{box}}$ around your final solution to the problem.
- Show your work. Demonstrate that you know how to get the correct answer, not just make a lucky guess.
- Clearly cross out any work that is incorrect.
- Partial credit will very rarely be awarded, if at all.
- If you run out of room, continue your work on the back of the previous page. Make a note that you've done this, and make it clear where your work continues.

(1) Do the parameterized lines $x = t - 1, y = 2t$ for $-\infty < t < \infty$ and $x = t, y = t + 1$ for $-\infty < t < \infty$ intersect? If so, where?

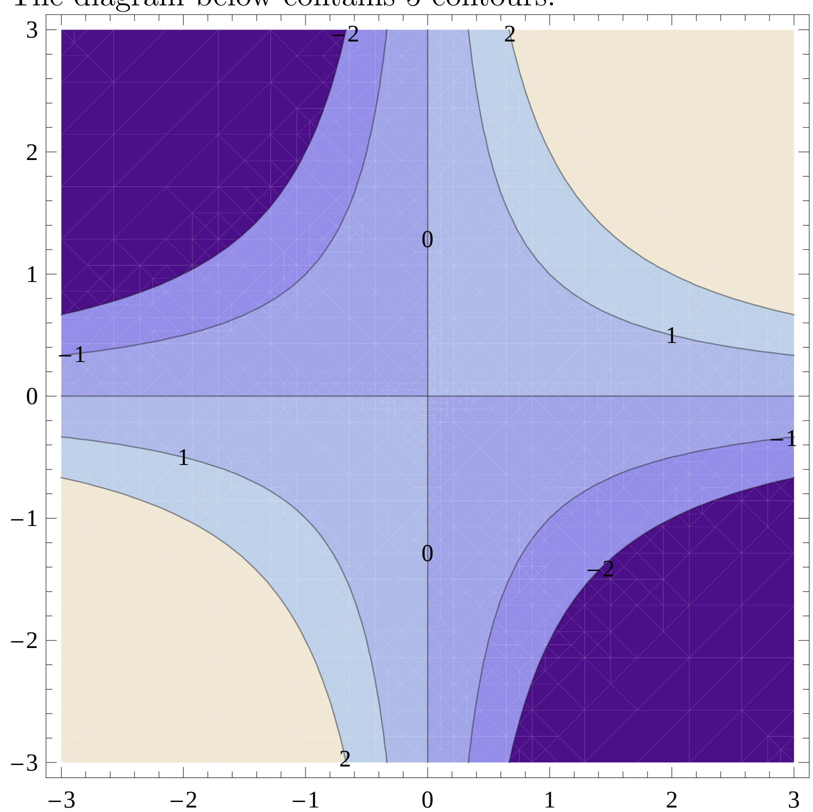
We're trying to solve the system of equations:

$$t - 1 = s$$
$$2t = s + 1$$

From these, we can conclude that $2t = t$, or $t = 0$. This gives us the point $(-1, 0)$.

(2) Sketch a contour diagram for the function $f(x, y) = xy$. Include at least 3 contours, and be sure to label them.

The diagram below contains 5 contours:



(3) Find the cosine of the angle between the vectors $(1, 0, 2)$ and $(1, 1, 1)$. Your final answer should not have any vector operations in it, but does not need to be otherwise simplified.

The cosine of the angle between two vectors $\vec{v}$ and $\vec{w}$ is $\frac{\vec{v}\cdot\vec{w}}{||\vec{v}||||\vec{w}||}$. Plugging in our values, we get:

$$\cos\theta = \frac{(1)(1) + (0)(1) + (2)(1)}{\sqrt{1^2 + 0^2 + 2^2}\sqrt{1^2 + 1^2 + 1^2}} = \frac{3}{\sqrt{5}\sqrt{3}}$$

(4) Find the area of the parallelogram formed by the points $(0, 0, 0)$, $(1, 0, 1)$, $(1, 1, 0)$, and $(2, 1, 1)$. Your final answer should not have any vector operations in it, but does not need to be otherwise simplified.

We're looking for $||(1, 0, 1)\times(1, 1, 0)||$. We can compute this using the matrix determinant method, or distributivity.

$$(\vec{i} + \vec{k}) \times (\vec{i} + \vec{j}) = \vec{i} \times \vec{i} + \vec{i} \times \vec{j} + \vec{k} \times \vec{i} + \vec{k} \times \vec{j} = 0 + \vec{k} + \vec{j} - \vec{i}$$

This vector has length $\sqrt{3}$.

(5) Parameterize the line segment from $(3,4,5)$ to $(3,3,3)$. Any complete parameterization with the correct direction will do.

$$(3, 4-t, 5-2t) \qquad \text{for } 0 \leq t \leq 1$$

(6) Find a vector in the direction of the tangent line to the parameterized curve $(\cos t, \sin t, 2t)$ at the point $(1,0,0)$.

If we let $\vec{r}(t) = (\cos t, \sin t, 2t)$, we're looking for $\vec{r}\,'(t)$ at the $t$ such that $\vec{r}(t) = (1,0,0)$. This happens at $t = 0$.

$$\vec{r}\,'(0) = (-\sin t, \cos t, 2)|_{t=0} = (0,1,2).$$

(7) Find the equation of the tangent plane to the graph of $z = x^2 + 2xy$ at the point $x = 1, y = 2$. Any form of the equation is acceptable.

$f(1,2) = 5 \qquad f_x(1,2) = 2x + 2y|_{(x,y)=(1,2)} = 6 \qquad f_y(1,2) = 2x|_{(x,y)=(1,2)} = 2$

Plugging into the equation for a tangent plane:

$$L(x,y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$$

We get:

$$L(x,y) = 5 + 6(x-1) + 2(y-2)$$

(8) Consider the function $f(x,y) = x^y$ at the point $x = 1, y = 1$. In what direction is $f$ increasing the fastest? Specifically, find a vector in this direction.

We're looking for $\nabla f(1,1)$.

$$f_x(1,1) = yx^{y-1}|_{(x,y)=(1,1)} = 1$$
$$f_y(1,1) = x^y \ln y|_{(x,y)=(1,1)} = 0$$

So $\nabla f(1,1) = (1,0)$.

# Math 2130 Prelim 1 (Spring 2017)

Name: ______________________________ Netid: ______________

Section (check which one applies):

☐ Discussion 1 (9:05am-9:55am)

☐ Discussion 2 (10:10am-11:00am)

| Do not write in this box | |
|---|---|
| (1) | ________ |
| (2) | ________ |
| (3) | ________ |
| (4) | ________ |
| (5) | ________ |
| (6) | ________ |
| (7) | ________ |
| (8) | ________ |
| Total | ________ |