Math 2130 Prelim 1 (Spring 2017)

Before the exam:
- Do not write anything on this page.
- Do not open the exam.
- Turn off your cell phone.
- Make sure your books, notes, and electronics are not visible during the exam.
- Do not wear headphones during the exam.

When you open your exam:
- Make sure your exam has all its pages. There are 6 pages, including the last, and 8 problems.
- If you believe there is a printing error, let me know right away.
- Write your name on the last page, and put a check in the box corresponding to your section.

During the exam:
- Do not talk or ask questions. If you are unsure what a question is asking, demonstrate your understanding as best you can.
- Be respectful of your fellow classmates.
- You may use the bathroom during the exam, but please ask first so I can keep track of who is out of the room at any one time.
- If you finish your exam before 2:00, you may leave early: hand your exam in at the front of the room, and do not discuss the exam directly outside the classroom. If you finish after 2:00, please remain quiet and seated until 2:15.

Notes on grading:
- Draw a box around your final solution to the problem.
- Show your work. Demonstrate that you know how to get the correct answer, not just make a lucky guess.
- Clearly cross out any work that is incorrect.
- Partial credit will very rarely be awarded, if at all.
- If you run out of room, continue your work on the back of the previous page. Make a note that you’ve done this, and make it clear where your work continues.
(1) Do the parameterized lines $x = t - 1, y = 2t$ for $-\infty < t < \infty$ and $x = t, y = t + 1$ for $-\infty < t < \infty$ intersect? If so, where?

We’re trying to solve the system of equations:

\[
\begin{align*}
    t - 1 &= s \\
    2t &= s + 1
\end{align*}
\]

From these, we can conclude that $2t = t$, or $t = 0$. This gives us the point $(-1, 0)$.

(2) Sketch a contour diagram for the function $f(x, y) = xy$. Include at least 3 contours, and be sure to label them.

The diagram below contains 5 contours:
(3) Find the cosine of the angle between the vectors $(1, 0, 2)$ and $(1, 1, 1)$. Your final answer should not have any vector operations in it, but does not need to be otherwise simplified.

The cosine of the angle between two vectors $\vec{v}$ and $\vec{w}$ is $\frac{\vec{v} \cdot \vec{w}}{|\vec{v}||\vec{w}|}$. Plugging in our values, we get:

$$\cos \theta = \frac{(1)(1) + (0)(1) + (2)(1)}{\sqrt{1^2 + 0^2 + 2^2} \sqrt{1^2 + 1^2 + 1^2}} = \frac{3}{\sqrt{3}\sqrt{5}}$$

(4) Find the area of the parallelogram formed by the points $(0, 0, 0)$, $(1, 0, 1)$, $(1, 1, 0)$, and $(2, 1, 1)$. Your final answer should not have any vector operations in it, but does not need to be otherwise simplified.

We’re looking for $||(1, 0, 1) \times (1, 1, 0)||$. We can compute this using the matrix determinant method, or distributivity.

$$(\vec{i} + \vec{k}) \times (\vec{i} + \vec{j}) = \vec{i} \times \vec{i} + \vec{i} \times \vec{j} + \vec{k} \times \vec{i} + \vec{k} \times \vec{j} = 0 + \vec{k} \times \vec{j} + \vec{i} \times \vec{j} = 0 + \vec{k} + \vec{j} - \vec{i}$$

This vector has length $\sqrt{3}$. 

(5) Parameterize the line segment from (3, 4, 5) to (3, 3, 3). Any complete parameterization with the correct direction will do.

\[(3, 4 - t, 5 - 2t) \quad \text{for } 0 \leq t \leq 1\]

(6) Find a vector in the direction of the tangent line to the parameterized curve \((\cos t, \sin t, 2t)\) at the point \((1, 0, 0)\).

If we let \(\vec{r}(t) = (\cos t, \sin t, 2t)\), we’re looking for \(\vec{r}'(t)\) at the \(t\) such that \(\vec{r}(t) = (1, 0, 0)\). This happens at \(t = 0\).

\[\vec{r}'(0) = (-\sin t, \cos t, 2)|_{t=0} = (0, 1, 2)\].
(7) Find the equation of the tangent plane to the graph of \( z = x^2 + 2xy \) at the point \( x = 1, y = 2 \). Any form of the equation is acceptable.

\[
\begin{align*}
f(1,2) &= 5 & f_x(1,2) &= 2x + 2y|_{(x,y)=(1,2)} = 6 & f_y(1,2) &= 2x|_{(x,y)=(1,2)} = 2 \\
\text{Plugging into the equation for a tangent plane:} & \\
L(x,y) &= f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2) \\
\text{We get:} & \\
L(x,y) &= 5 + 6(x-1) + 2(y-2)
\end{align*}
\]

(8) Consider the function \( f(x,y) = x^y \) at the point \( x = 1, y = 1 \). In what direction is \( f \) increasing the fastest? Specifically, find a vector in this direction.

We’re looking for \( \nabla f(1,1) \).

\[
\begin{align*}
f_x(1,1) &= yx^{y-1}|_{(x,y)=(1,1)} = 1 \\
f_y(1,1) &= x^y \ln y|_{(x,y)=(1,1)} = 0 \\
\text{So } \nabla f(1,1) &= (1,0).
\end{align*}
\]
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Name: ____________________________________________ Netid: __________________________
Section (check which one applies):
☐ Discussion 1 (9:05am-9:55am)
☐ Discussion 2 (10:10am-11:00am)

Do not write in this box

(1) ________
(2) ________
(3) ________
(4) ________
(5) ________
(6) ________
(7) ________
(8) ________

Total ________