Math 2130 Homework 5: 14.6-14.7

(1) A point located at distance r from the origin with angle θ counter-clockwise from the x-axis has x coordinate $r\cos\theta$. Suppose r and θ are functions of t. Write out a chain rule for $\frac{dx}{dt}$. Suppose r is increasing at a constant rate of one unit per second, and θ is increasing at a constant rate of one unit per second (spiralling away from the origin). Use your chain rule to write $\frac{dx}{dt}$ as a function of r and θ. (Normally you would write it in terms of x and y.)

(2) The temperature in a room at position x, y, z relative to a heat source is given by $H(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$. A fly is flying around in the helix $(\cos t, \sin t, t)$ for $-2\pi \leq t \leq 2\pi$. Work out, using the multivariable chain rule, the rate of change in temperature of the fly at $t = \pi$.

(3) The temperature in a room at position x, y, z relative to a heat source is given by $H(x, y, z) = (x^2 + y^2 + z^2)^{-\frac{1}{2}}$. A fly is flying around in the helix $(\cos t, \sin t, t)$ for $-2\pi \leq t \leq 2\pi$. Work out, by plugging in the formulas for $x(t), y(t)$, and $z(t)$ into the equation for H to directly get H as a function of t, the rate of change in temperature of the fly at $t = \pi$.

(4) Verify that $f_{xy} = f_{yx}$ for $f(x, y) = x^y$

(5) Use a quadratic approximation for $f(x, y) = x^y$ around $(x, y) = (3, 2)$ to approximate $3.02^{1.99}$.