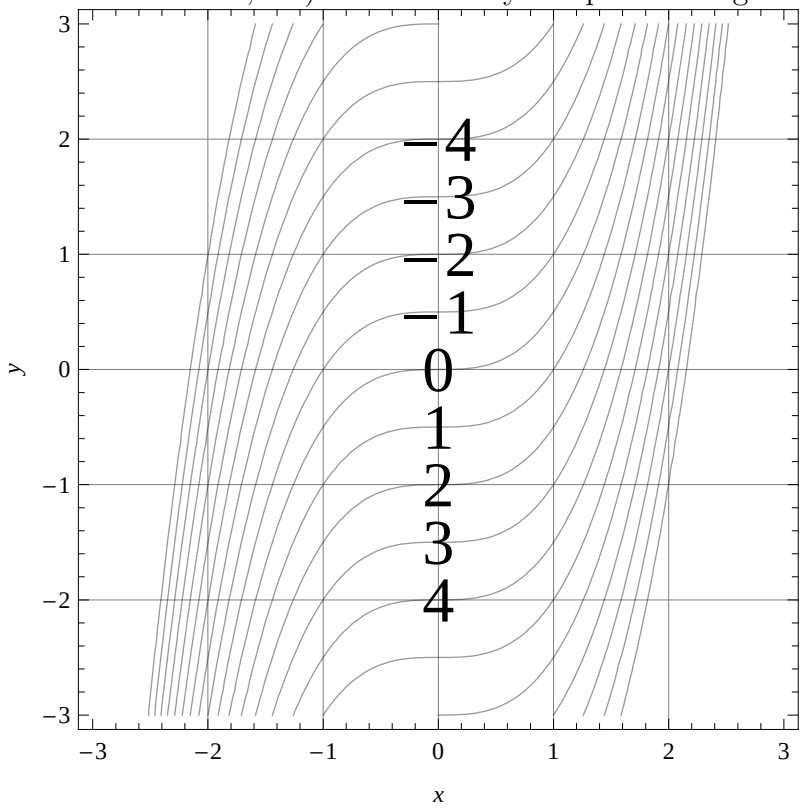
Math 2130 Homework 4: 14.3, 14.4, 14.5

(1) Shown below is the contour plot of a function $f(x, y)$. In what directions do the gradient vectors at $(0, 0), (1, 0)$ and $(-2, -1)$ point (eg. up, up and to the right, down and to the left, etc)? How do they compare in magnitude?



(2) Find the equation for the tangent plane to the graph $z = f(x, y) = \sqrt{49 - x^2 - y^2}$ at $x = 3, y = 2$.

(3) Find the equation for the linear approximation to the function $f(x, y, z) = xyz$ at $x = 1, y = 2, z = 3$.

(4) Use a linear approximation to $f(x, y) = x^y$ at $x = 3, y = 2$ to approximate $3.02^{1.99}$. Use a calculator to compare your answer and the actual value.

(5) If $f(x, y) = xy^2$, what is df?

(6) If $f(x, y) = \frac{x^2}{y}$, compute ∇f.

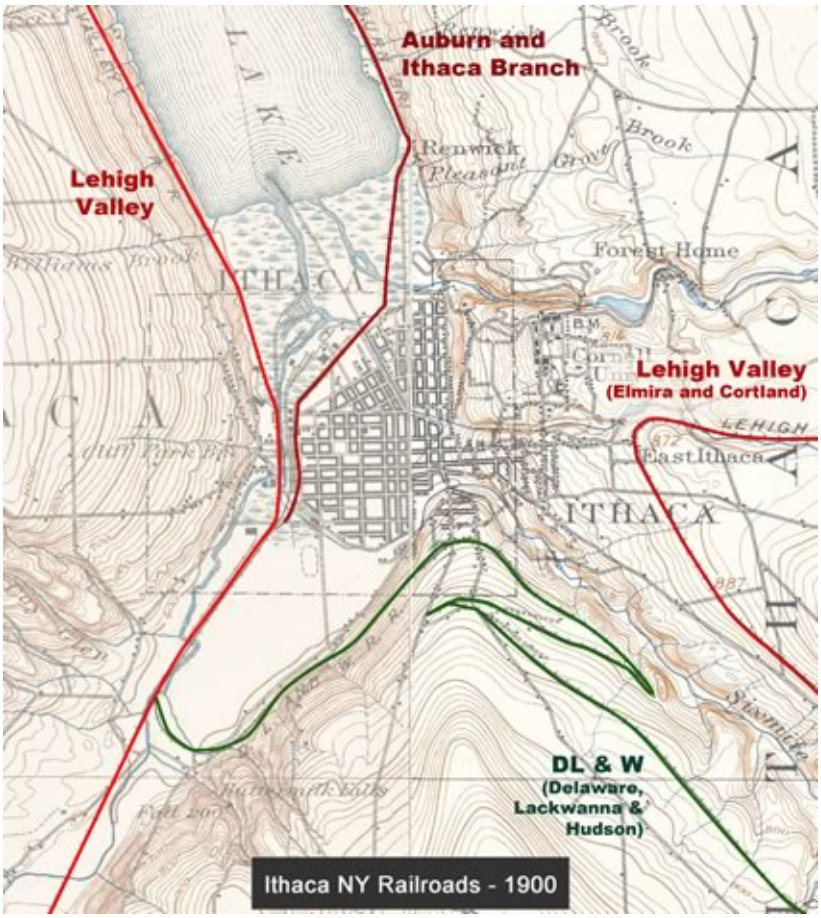
(7) If $f(x, y, z) = \frac{1}{x^2+y^2+z}$ compute $\nabla f(1, 2, 3)$.

(8) Suppose the function $f(x, y)$ has level curve $y = x^2$. Sketch the graph of $y = x^2$ and draw out possible gradient vectors for $f(x, y)$ at 5 points along the curve.

(9) The gradient vectors of a function $f(x, y, z)$ all point directly towards the origin. What do the level surfaces of $f(x, y, z)$ look like? Which correspond to larger values of f?

(10) Shown below is a map of railroad lines through Ithaca in 1900.

Notice that the Delaware, Lackawana & Hudson line (now the South Hill Recreation Way) zig-zags down the hill to avoid being too steep. Suppose you're deciding the trail for a train line. Describe how to choose the orientation of the line in terms of the slope of the hill, the direction the hill is sloped in, and the desired slope of the track.


Auburn and
Ithaca Branch
Lehigh
Valley
Lehigh Valley
(Elmira and Cortland)
DL & W
(Delaware,
Lackwanna &
Hudson)
ITHACA
Forest Home
East Ithaca
Renwick
Pleasant
Brook
Ithaca NY Railroads - 1900