Math 2130 Homework 3: 17.1, 17.2, 14.1, 14.2

1. Parameterize the circle in the $xz$ plane of radius 3 centered at $(0, 0, 1)$.

   $$(3 \cos t, 0, 3 \sin t + 1) \text{ for } 0 \leq t \leq 2\pi.$$  

2. Parameterize the line segment from the point $(1, 2, 3)$ to the point $(3, 2, 1)$.

   $$(1 + 2t, 2, 3 - 2t) \text{ for } 0 \leq t \leq 1.$$  

3. Describe precisely the curve parameterized by the function $r(t) = (e^t, (e^t)^2)$ from $t = 0$ to $t = 1$.

   This is the portion of the graph of $y = x^2$ on the interval $[1, e]$.

4. Do the curves parameterized by $v(t) = (t^2, t + 1)$ from $t = 0$ to $t = 4$ and $w(t) = (t, t - 1)$ from $t = 0$ to $t = 4$ intersect? If so, where?

   We’re solving the system of equations $t^2 = s$ and $t + 1 = s - 1$. Note the new variable. We can conclude that $(s - 2)^2 = s$, or that $s^2 - 4s + 4 = s$ or $s^2 - 5s + 4 = 0$ which has solutions at $s = \frac{5 \pm \sqrt{25-16}}{2}$ which is either $s = 1$ (in which case $t = -1$) and $s = 4$ (in which case $t = 2$). Note that since $t$ is not in the specified interval in the first case, it is not a point of intersection. So the only intersection is when $t = 2$ and $s = 4$, which is at the point $(4, 3)$.

5. If two particles follow the paths in the above problem, will they collide? Will they cross paths?

   They will cross paths, since that is the same as their paths intersecting, but since they intersect at different values of the time parameter, they will not collide.

6. You have a vertically-oriented helical spring and shine a light straight down the central axis (imagine the light source is so far away that the light beams are perfectly vertical). What will its shadow look like? What if you shine light on it from the side (perpendicular to the central axis with perfectly horizontal light beams)?

   A circle and a sinusoid.

7. If the position of a particle at time $t$ is given by $r(t) = (e^t, \sin(t))$:  
   
   (a) What is its velocity vector at time $t$? $$(e^t, \cos(t))$$  
   
   (b) What is its velocity vector at time $t = 0$? $$(1, 1)$$  
   
   (c) Parameterize the tangent line to the curve at $t = 0$. $$(1, 0) + t(1, 1) \text{ for } -\infty < t < \infty$$  
   
   (d) What is its acceleration vector at time $t = 0$? $$(e^t, -\sin(t))|_{t=0} = (1, 0)$$
(8) Let $f(x, y) = x^y$. Compute $f_x(x, y)$ and $f_y(x, y)$.

$$f_x = yx^{y-1}, \quad f_y = x^y \ln(x)$$

(9) Knowing that $f_x(1, 2) > 0$ tells you something specific about a specific cross section of the graph of $f$. Which cross section, and what does it tell you?

It tells you that the cross section through the plane $y = 2$ has positive slope at $x = 1$.

(10) Shown below is a contour diagram of a function $f(x, y)$. Approximate $f_y(0, 0)$.

$$f(0.5, 0) - f(0, 0) = -1 - 0 = -2$$

(11) Let $f(x, y) = 2xy$. Compute $f_x(2, 3)$ and $f_y(2, 3)$.

6 and 4