Math 2130 Homework 14: 20.1-20.3, 21.2

(1) Which of the following expressions make sense? Which of them are always constant 0 or $\vec{0}$? Which do not make sense? Assume everything is the three dimensional version for this problem.
   (a) curl(grad(f)).
   (b) grad(div($\vec{F}$)).
   (c) div(curl(grad(f))).
   (d) div(grad(f)).
   (e) div(curl(f)).
   (f) curl(curl($\vec{F}$)).
   (g) div(div($\vec{F}$)).

(2) A vector field $\vec{F}$ satisfies that everywhere on the side surface of a unit cylinder (radius 1, height 1), curl($\vec{F}$) is pointing directly outward perpendicular to the cylinder, and has length 3. The circulation of $\vec{F}$ counterclockwise around the top edge (the curve where the side surface meets the top surface) of the cylinder is 4. What is the circulation of $\vec{F}$ measured counterclockwise around the bottom edge of the cylinder?

(3) Verify Stokes' Theorem for the vector field $\vec{F} = (y, xz, x^2)$ over the triangle with vertices $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$.

(4) Verify that the Jacobian of the map $x = \sqrt{2r}\cos\theta$, $y = \sqrt{2r}\sin\theta$ is constant 1.