Math 2130 Homework 14: 20.1-20.3, 21.2

(1) Which of the following expressions make sense? Which of them are always constant 0 or \( \vec{0} \)? Which do not make sense? Assume everything is the three dimensional version for this problem.
(a) \( \text{curl}(\text{grad}(f)) \).
(b) \( \text{grad}(\text{div}(\vec{F})) \).
(c) \( \text{div}(\text{curl}(\text{grad}(f))) \).
(d) \( \text{div}(\text{grad}(f)) \).
(e) \( \text{div}(\text{curl}(f)) \).
(f) \( \text{curl}(\text{curl}(\vec{F})) \).
(g) \( \text{div}(\text{div}(\vec{F})) \).

(2) A vector field \( \vec{F} \) satisfies that everywhere on the side surface of a unit cylinder (radius 1, height 1), \( \text{curl}(\vec{F}) \) is pointing directly outward perpendicular to the cylinder, and has length 3. The circulation of \( \vec{F} \) counterclockwise around the top edge (the curve where the side surface meets the top surface) of the cylinder is 4. What is the circulation of \( \vec{F} \) measured counterclockwise around the bottom edge of the cylinder?

(3) Verify Stokes’ Theorem for the vector field \( \vec{F} = (y, xz, x^2) \) over the triangle with vertices \((1,0,0),(0,1,0),(0,0,1)\).

(4) Verify that the Jacobian of the map \( x = \sqrt{2r} \cos \theta, y = \sqrt{2r} \sin \theta \) is constant 1.