Math 2130 Homework 11: 18.4, Divergence Theorem

(1) What does the Curl Test tell us about the following vector fields (path-independent, path-dependent, or does not tell us anything):

  (a) $\vec{F}(x,y) = (-3x^2y^2 + x^7, 2x^3y - 10y + 4)$

  (b) $\vec{F}(x,y) = \left(\frac{-y+x^2y+y^3}{x^2+y^2}, \frac{x+x^4+y^2x^2}{x^2+y^2}\right)$

  (c) $\vec{F}(x,y) = (2x^3y - 10y + 4, -3x^2y^2 + x^7)$

(2) Let C be the counterclockwise boundary of the square with vertices $(0,0), (0,1), (1,1)$, and $(1,0)$, and $\vec{F}(x,y) = (x-y, x+y)$. Use Green's Theorem to compute $\int_C \vec{F} \cdot d\vec{r}$. Appreciate that this saved you having to do 4 relatively easy vector line integrals.

(3) Verify the 2-Dimensional Divergence Theorem for $\vec{F}(x,y) = (x^2, y^2)$ around the counterclockwise boundary of the triangle with vertices $(0,0), (1,0)$ and $(0,1)$ by computing both sides of the equation directly.

(4) Let R be the half of the solid unit circle with $y \geq 0$. Let C_1 be the half of the unit circle with $y \geq 0$ (the upper half of the boundary of R) and let C_2 be the line from $(-1,0)$ to $(1,0)$ (the lower half of the boundary). Orient both counterclockwise around R. You know that:

$$\int_R [(F_2)_x - (F_1)_y] dA = 3 \qquad \int_{C_1} \vec{F} \cdot d\vec{r} = 4$$

Use Green's Theorem to determine $\int_{C_2} \vec{F} \cdot d\vec{r}$.