Math 2130 Spring 2013 Prelim 3

Each question is worth 25 points. Good luck.

1. Consider the double integral

\[ \int_0^a \int_0^{\sqrt{a^2 - y^2}} \left( x^2 + y^2 \right) \, dx \, dy. \]

a) (15 pts) Rewrite the double integral as an equivalent double integral in polar coordinates.
b) (10 pts) Evaluate the double integral.

2. Consider the vector field \( \vec{v} = x \vec{i} - y \vec{j} \).

a) (5 points) Sketch the vector field.
b) (5 pts) Sketch the flow line that passes through the point (1,1).
c) (15 pts) Find the parametric equations for the flow line \( \vec{r}(t) = x(t) \vec{i} + y(t) \vec{j} \) that passes through the point (1,1) at time \( t = 0 \).

3. A cylindrical hole of radius \( a \) is drilled symmetrically through a solid sphere of radius \( 2a \), as shown here.

Find the volume of the remaining solid, as follows:
a) (15 points) Express the volume of the remaining solid as a triple integral in cylindrical coordinates.
b) (10 points) Evaluate the triple integral.

4. Let \( C \) denote the plane curve \( \vec{r}(t) = (e^t \cos t) \vec{i} + (e^t \sin t) \vec{j} \), for \( 0 \leq t \leq 2\pi \).
a) (10 pts) Sketch the curve \( C \) in the \( xy \) plane.
b) (15 points) Evaluate the line integral along \( C \) given by

\[ \int_C \frac{xdx + ydy}{(x^2 + y^2)^{3/2}}. \]