Math 2130
Spring 2014
Prelim II
3/25/14
Time Limit: 90 Minutes

Name (Print): ______________________________

Discussion: ______________________________

TA: ______________________________

This exam contains 7 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use any electronics on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Do not simplify:** for example, $2(0.7) + \cos(\pi/5)$ is a good number.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	24	
4	16	
5	20	
Total:	100	

1. Consider the integral

$$\int_0^1 \int_y^1 e^{x^2} dxdy.$$

(a) (4 points) Sketch the region of integration. (Please label everything.)

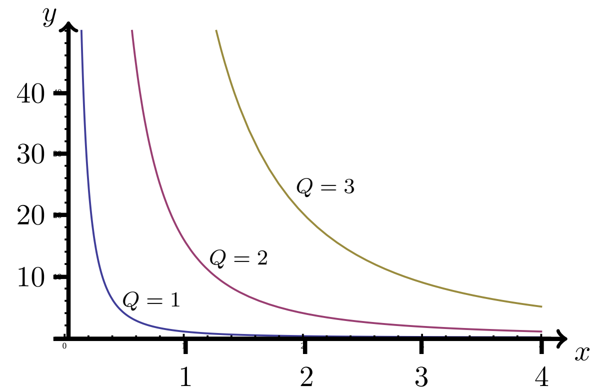
(b) (8 points) Use your sketch in (a) to reverse the order of integration.

(c) (8 points) Evaluate either the original integral or your answer in (b).

2. An industry manufactures a product from two raw materials under a budget constraint. The quantity produced, Q, is given by the formula

$$Q(x, y) = x^{1/2}y^{1/4}$$

The industry has a budget constraint given by $30x+4y \le 120$. The figure below shows contours labeled with values of $Q(x, y)$. Let R be the region given by $x \ge 0$, $y \ge 0$, $30x + 4y \le 120$.



(a) (4 points) Use the contours to discuss why there appears to be no local maximums of $Q(x, y)$ inside of R, i.e. where $x > 0$, $y > 0$, and $30x + 4y \le 120$.

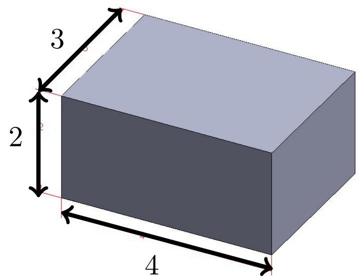
(b) (4 points) Verify the conclusion in part (a) by showing that $Q(x, y)$ does not have any critical points whenever $x > 0$ and $y > 0$.

(c) (4 points) Draw the constraint $30x + 4y \leq 120$ on the graph. Use your graph to mark where there appears to be a maximum of $f(x, y)$ over the region R. Explain your reasoning.

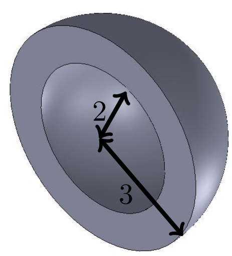
(d) (8 points) Precisely determine the point in part (b) by using the method of Lagrange multipliers to find the maximum of $f(x, y)$ subject to the constraint $30x + 4y \leq 120$ between the points (0,30) and (4,0).

3. In parts (a)–(c), choose coordinates; set up a triple integral, including limits of integration, for a density function $f(x, y) = 2$ over the region; and evaluate the integral.

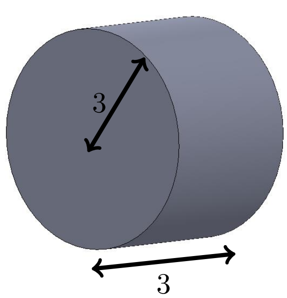
(a) (8 points)



(b) (8 points)



(c) (8 points)



4. (a) (4 points) Choose coordinates and write a double integral for a function p over the region R given by $x^2 + y^2 \leq 100$, $x \geq 0$, $y \geq 0$.

   (b) (4 points) Use your answer in (a) to find k such that

$$p(x, y) = \begin{cases} kx & (x, y) \text{ in } R \\ 0 & \text{otherwise} \end{cases}$$

   is a probability density function.

   (c) (8 points) Find the probability that a point chosen in R according to the probability density in part (b) is less than 10 units from the origin.

5. In the following problems, please briefly explain what is wrong with the statement. Cite any relevant theorems.

   (a) (4 points) If the partial derivatives $f_x(0,0)$ and $f_y(0,0)$ both exist, then $f(x,y)$ is differentiable at the origin.

   (b) (4 points) There is a point (x,y) where $f(x,y) = x + y$ is not differentiable.

   (c) (4 points) If $f_x = f_y = 0$ at $(1,1)$, then f has a local maximum or local minimum at $(1,1)$.

   (d) (4 points) A function having no critical points in a region R cannot have a global maximum in the region.

   (e) (4 points) It is possible that a continuous function $f(x,y)$ on R given by $0 \leq x^2 + y^2 \leq 1$ does not have a global minimum on R.