

Math 2130
Spring 2014
Prelim I
2/20/14

Name (Print): _____

Discussion: _____

Time Limit: 90 Minutes

TA: _____

This exam contains 7 pages (including this cover page) and 6 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use any electronics on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Do not simplify:** for example, $2(0.7) + \cos(\pi/5)$ is a good number.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit.** A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Problem	Points	Score
1	20	
2	15	
3	20	
4	15	
5	10	
6	15	
Total:	95	

Do not write in the table to the right.

1. Calculate the indicated derivatives.

(a) (5 points) $f_x(x, y)$ if $f(x, y) = x \cos(xy) - 2y$.

(b) (5 points) $f_y(x, y)$ if $h(x, y) = x^3 - xy^2$.

(c) (5 points) $\nabla h(2, 4)$ if $h(x, y) = x^3 - xy^2$.

(d) (5 points) The directional derivative of $h(x, y) = x^3 - xy^2$ at the point $(2, 4)$ in the direction of $\vec{i} + \vec{j}$.

2. Suppose the force on an object is $\vec{F} = -20\vec{j}$ and we are given a vector $\vec{v} = 2\vec{i} + 3\vec{j}$. Calculate the following:

(a) (5 points) The component of \vec{F} parallel to \vec{v} .

(b) (5 points) The component of \vec{F} perpendicular to \vec{v} .

(c) (5 points) The work, W , done by force \vec{F} through displacement \vec{v} .

3. Consider points $A = (-1, 2, 0)$, $B = (3, 3, 3)$, and $C = (1, -1, 5)$.
- (a) (5 points) Resolve displacement vectors \overrightarrow{AB} and \overrightarrow{AC} into components, where \overrightarrow{AB} denotes the vector *from* point A to point B and similarly for \overrightarrow{AC} .
- (b) (5 points) Calculate $\overrightarrow{AB} \times \overrightarrow{AC}$.
- (c) (5 points) Find an equation for the plane that passes through these three points.
- (d) (5 points) Find the area of the triangle determined by these points.

4. In this problem, consider $f(x, y) = x^2 + y^2$ and the surface given by $z = f(x, y)$.
- (a) (5 points) Draw the surface $z = f(x, y)$ and the point $(-1, 1, 2)$ on the surface.
- (b) (5 points) Find an equation for the tangent plane to the surface at $(-1, 1, 2)$.
- (c) (5 points) Use your answer in part (b) to approximate $f(-1.1, 0.9)$.

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5. The current in a river is pushing a boat in direction 30° north of east with a speed of 12 km/hr. The wind is pushing the same boat in a direction 70° south of east with a speed of 7 km/hr.
- (a) (5 points) Draw vectors representing the velocity vectors of the current and wind. (Please specify directions North, South, East and West, as well as the x and y directions.)
- (b) (5 points) Find the velocity vector of the boat's engine (relative to the water) if the boat actually moves due east at a speed of 30 km/hr relative to the ground.

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6. You are standing at the point $(1, 1, 3)$ on the hill whose equation is given by $z = 5y - x^2 - y^2$. Suppose North points in the positive y -direction and East points in the positive x -direction.
- (a) (5 points) If you choose to climb in the direction of steepest ascent, what is your initial rate of ascent in this direction?
- (b) (5 points) If you decide to go straight northwest, will you be ascending or descending? At what rate?
- (c) (5 points) If you decide to maintain your altitude, in what directions can you go?