Math 2130: Final Exam (May 18, 2015)

Your Name: **Your Netid:**

Your Section Number and/or Time:

This exam has 14 pages with 10 problems adding up to 200 points.
You have 150 minutes.
If you need extra space, you can use the other side of each page or the last mostly blank page.
No calculators or books are allowed.

Please show all of your work (including the intermediate steps). Writing clearly and legibly will improve your chances of receiving the maximum credit that your solution deserves. Good luck!

Problem #	Max pts	Earned
1	20	
2	20	
3	25	
4	15	
5	25	
6	20	
7	20	
8	15	
9	15	
10	25	
Total	200	

Academic Integrity is expected of all students of Cornell University at all times, whether in presence or absence of members of the faculty.

Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Signature of the Student: ______________________________

Unless specifically requested, answers on this exam NEED NOT be simplified or completely numerical. Arithmetic expressions are fine. Ordinarily, answers like

$$\ln\left(\frac{10+2}{5}\right), \qquad \frac{3x^2(x^4-1)-4x^3(x^3+1)}{(x^4-1)^2}, \qquad \frac{1}{2}\left(4x^3\right)\left(x^4+1\right)^{.5}$$

are completely acceptable.

Problem 1) (20 Points)

1a) Find a parametrization of the line segment (or just the equation of the line) between the points $(1,2,3)$ and $(4,6,7)$ in R^3.

1b) Let $\mathcal{C}$ be the line segment described in part a), oriented as starting from $(4,6,7)$ and ending at $(1,2,3)$. Use your parametrization in part a) to calculate the value of the line integral

$$\int_{\mathcal{C}} y\ dx + z\ dy - 2x\ dz.$$

Problem 2) (20 Points) Use a systematic procedure to find a function $f(x, y, z)$ whose gradient is the vector field

$$(yz^2e^{xz^2},\ e^{xz^2} - z\ \sin(yz),\ 2xyze^{xz^2} +\ 4\ - y\ \sin(yz))$$

Problem 3) (25 Points) Let

$$\begin{aligned} x = &\quad u^2 - v^2 \\ y = &\quad 2uv. \end{aligned}$$

Suppose $z = f(x, y)$ is a differentiable function.

3a) Express

$$\frac{\partial z}{\partial u}$$

in terms of

$$\frac{\partial z}{\partial x}, \quad \frac{\partial z}{\partial y}, \quad u, \text{ and } v.$$

3b) Show that

$$\frac{1}{4(u^2+v^2)}\left[\left(\frac{\partial z}{\partial u}\right)^2+\left(\frac{\partial z}{\partial v}\right)^2\right]=\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2.$$

Problem 4) (15 Points) Reverse the order of integration to find the value of

$$\int_0^1 \int_y^1 \cos x^2 \; dx \; dy.$$

Problem 5) (25 Points) Set up as an integral or sum of integrals the double integral of the function $e^{x^2y^2}$ over the trapezoid in R^2 with vertices $(0,0)$, $(3,0)$, $(5,2)$, and $(1,1)$. Include all limits of integration but do not attempt to compute the value of the double integral.

Problem 6) (20 Points) Consider the (solid) region in R^3 bounded by the four planes $z = 0,\ x = 1,\ y = 2,$ and $x + 2y + 3z = 6.$

Let $f(x, y, z)$ be a differentiable function within this region. Set up the triple integral of $f(x, y, z)$ over this region. Include all limits of integration but do not attempt to compute the value of the integral.

Problem 7) (20 Points) Find the maximum and minimum values attained by the function $f(x, y) = x^2 - 2x + 3 + y^2 - 6y$ at points of the rectangular region in the xy plane with vertices $(-2, 2)$, $(4, 2)$, $(4, 4)$, and $(-2, 4)$.
(Please be sure and show all the needed systematic work to justify your answer.)

Problem 8) (15 Points) Using the method of separation of variables, find a solution of the form $u(x, y) = X(x)Y(y)$ to the partial differential equation

$$u_x = u_y + 2u$$

which also satisfies $u(x, 0) = e^{5x}$.

Problem 9) (15 Points) The method of separation of variables applied to a certain partial differential equation (with boundary conditions) defined for $0 \leq x \leq 1$ produces the series solution

$$u(x,t) = \frac{a_0}{2} + \Sigma_{n=1}^{\infty} \cos(n\pi x)\left[a_n \cos(4n\pi t) + b_n \sin(4n\pi t)\right]$$

with orthogonality relation

$$\int_0^1 \cos(n\pi x)\cos(m\pi x)\ dx = 0$$

when n and m are unequal non-negative whole numbers.
(Take this as given - you don't have to verify it.)

It is desired to fit the initial conditions $u(x,0) = f(x)$ and $u_t(x,0) = g(x)$. Use the given orthogonality relations to obtain formulas for the coefficients a_0 as well as for the a_k and b_k when $k = 1, 2, \ldots$ in the above solution. Of course your answer will be in terms of the functions $f(x)$ and $g(x)$.

Problem 10) (25 Points) Consider the wave equation

$$u_{xx} = \frac{1}{9} u_{tt}$$

for a function $u(x,t)$ defined when $0 \leq x \leq 2\pi$ and satisfying the boundary conditions $u_x(0,t) = 0$, $u(2\pi,t) = 0$.

10a) Using the method of separation of variables, find (up to constant factors) all nontrivial functions $X(x)$ and $T(t)$ so that $u(x,t) = X(x)T(t)$ satisfies the above equation and $X(x)$ satisfies the boundary conditions $X'(0) = 0$ and $X(2\pi) = 0$.

10b) Using the full collection of solutions generated by the separation of variables procedure in part 10a), write down a general series solution satisfying the above equation with these boundary conditions.

10c) Write down the orthogonality relations you would expect to hold for the functions $X(x)$ you found in part 10a).

More space - Be sure and indicate clearly on the original problem page if you are continuing some problem on this page.