1. (20 pts) Let \( \mathbf{F} = (y, -x, z) \) be a vector field in 3-space.
   
   (a) Calculate the line integral \( \int_{C} \mathbf{F} \cdot d\mathbf{s} \), where \( C \) is the curve given by the path 
   \( \mathbf{r}(t) = (\cos t, 2\sin t, t) \), for \( 0 \leq t \leq \pi \).
   
   (b) Calculate the line integral \( \int_{C} \mathbf{F} \cdot d\mathbf{s} \), where \( C \) is the line segment from \((1, 1, 1)\) to 
   \((1, -1, 2)\).

2. (15 pts) A syzygy in astronomy is a straight line configuration of three heavenly bodies 
   (such as a lunar or solar eclipse, involving the earth, moon and sun). Consider three 
   points, whose position at time \(-\infty < t < \infty\) is given by the vector functions
   
   \( \mathbf{r}_1(t) = 4\mathbf{k} + t\mathbf{k}, \quad \mathbf{r}_2(t) = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + t\mathbf{j}, \quad \mathbf{r}_3(t) = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + t\mathbf{i} \).

   Find the value of \( t \) when the three points above form a syzygy, namely when they are 
   all in a straight line.

3. (15 pts) A fighter plane, which can shoot a laser beam straight ahead, travels along the 
   path \( \mathbf{r}(t) = (2 + \cos t, \sin t) \). Show that there is precisely one time \( t \) for \( 0 \leq t < 2\pi \) at 
   which the pilot can hit a target located at the point \((4, 0)\). Calculate what that time 
   \( t \) is.

4. (15 pts) Calculate the limits or show that they do not exist. Remember to justify your 
   answers.

   \[ \lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2 + y^2} \quad \text{(a)} \quad \lim_{(x,y)\to(0,0)} \frac{x^2\sqrt{|y|}}{x^2 + y^2} \quad \text{(b)} \]
5. (20 pts) A new asteroid called Oyd has been discovered. It is an ellipsoid given by the equation \( x^2 + y^2 + 2z^2 = 1 \). Astronauts have just landed at the point \((\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\) and the sun is in the direction \((-1, 4, -1)\).

(a) Calculate the gradient of \( x^2 + y^2 + 2z^2 \) and use it to determine an equation of the tangent plane at the point \((\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\).

(b) Are the astronauts in sunlight or are they in darkness? Justify your answer.

(c) The astronauts want to travel along a curve on the surface of Oyd from \((\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\) such that they stay the same distance from the center of Oyd. Find a parametrization of that curve, starting and ending at \((\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\), which is the intersection of the sphere with center at the origin through \((\frac{1}{2}, \frac{1}{2}, \frac{1}{2})\) and the surface of Oyd. Remember the parametrization includes bounds on the parameters.

6. (15 pts) The figure below shows some level curves of a differentiable function \( f(x, y) \) which are nested circles on each side of the central line, where the value of \( f \) decreases from each level to the next going from the right inner circle to the left inner circle as shown.

(a) Sketch a reasonable graph of the function \( f \) along the line through the points \(A\) and \(B\) in the plane, including maxima, minima and asymptotes.

(b) What is the angle between the gradient of \( f \) at \(A\), and the gradient of \( f \) at \(B\)? Justify your answer.

(c) Consider the directional derivative, \( D_u f(C) \), at the point \(C\) in the direction \(u\). Is \( D_u f(C) \) positive, negative, or zero? Justify your answer.