MATH 192 SUMMER 2007 PRELIM 2

Please write your name on all of the exam booklets you use. ***Show all your work*** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them in detail. No calculators are permitted. Good luck!

1. (8 pts) Consider the the function $f(x,y) = x^2 + xy + y^2$.

   (a) Find the directions in which $f(x,y)$ increases and decreases most rapidly at $P_0(-1,1)$.

   (b) Find the derivatives of $f(x,y)$ in these directions.

2. (7 pts) Find the linearization $L(x,y,z)$ of $f(x,y,z) = \dfrac{\sin(xy)}{z}$ at $(\dfrac{\pi}{2}, 1, 1)$.

3. (7 pts) Find the critical point(s) of $f(x,y) = x^2 + y^2 + xy + 1$ and determine their nature. Justify your answer.

4. (8 pts) Find the extrema of $x^2y^2z^2$ on the sphere $x^2 + y^2 + z^2 = 3$.

5. (7 pts) Evaluate the double integral $\displaystyle\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin(y)}{x+y}\, dy\, dx$.
   Hint: $\ln(a) - \ln(b) = \ln(a/b)$ $(a, b > 0)$.

6. (7 pts) Evaluate the double integral

$$\iint_R \sqrt{1 - x^2 - y^2}\, dA,$$

   where R is the unit disk centered at the origin.

7. (7 pts) Express the volume of the pyramid D with vertices $O(0,0,0)$, $A(1,0,0)$, $B(1,1,0)$ and $C(0,0,1)$ as a ***triple*** integral. Set up the limits of integration but **<u>do not evaluate</u>** the integral.

8. (7 pts) Evaluate the triple integral

$$\iiint_D \sqrt{x^2 + y^2 + z^2}\, dV,$$

   where D is the unit sphere centered at the origin.

9. (7 pts) Evaluate $\displaystyle\int_C (x^2 + y^2 + z^2)\, ds$, where C is the straight line segment from $(1,0,0)$ to $(1,3,0)$.