

Math 1920, Prelim 2
October 30, 2008, 7:30 PM to 9:00 PM

You are NOT allowed calculators, the text or any other book or notes. SHOW ALL WORK! Write your name and Lecture/Section number on each booklet you use. You may leave when you have finished, but if you have not handed in your exam booklet and left the room by 8:45, please remain in your seat so as not to disturb others who are still working.

- 1) a) (7 points) Find the directions in which the function $f(x, y, z) = 2xy - yz$ increases and decreases most rapidly at the point $P_0(1, -1, 1)$. (Recall that a direction in three-dimensional space is specified by a unit vector.)
b) (7 points) Calculate the directional derivatives of f in these directions.
c) (6 points) Is there a direction at P_0 in which the derivative of f is -3 ? Provide a reason for your answer.

- 2) a) (14 points) Calculate the standard linear approximation of the function $f(x, y, z) = xz - 3yz + 2$ at the point $(1, 1, 2)$.
b) (6 points) Use this to determine the approximate value of the function at the point $(1.1, 1.1, 2.1)$

- 3) Consider the region R inside the unit circle centered at the origin and outside the region defined by $-x < y < x$.
a) (5 points) Sketch R .
b) (7 points) Calculate the area of R .
c) (8 points) Find the centroid of R .

Note that the centroid (\bar{x}, \bar{y}) is given with the following equations:

$$\bar{x} = \frac{\iint_R x \, dA}{\iint_R dA} \quad \text{and} \quad \bar{y} = \frac{\iint_R y \, dA}{\iint_R dA}$$

- 4) (20 points) Consider the points $P_1(0, 0)$, $P_2(1, 0)$, $P_3(a, b)$ in the plane, a, b constants. Find the point $P(x, y)$ such that the sum of the squares of the distances from P to each of P_1, P_2, P_3 is minimized. Be careful to justify your answer.

- 5) (20 points) Find the maximum volume of a box determined by eight points $(\pm x, \pm y, \pm z)$ inside the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$.