Math 192, Prelim 2
October 25, 2007. 7:30-9:00

*You are NOT allowed calculators, the text, or any other book or notes. SHOW ALL WORK! Write your name and Lecture/Section number on each booklet you use*

1) Let the surfaces $S_1$, $S_2$ be defined by the equations $z = x^2 + 2y^2 + 2$ and $z = 5x - y - 2$, respectively.
a) (2 points) Is the point $(1, 0, 3)$ on these surfaces?
b) (7 points) Find an equation of the plane tangent to the surface $S_1$ at the point $(1, 0, 3)$.
c) (7 points) Find a parametric equation of the line tangent to the curve of intersection of $S_1$ and $S_2$ at the point $(1, 0, 3)$.

2) Consider the function $f(x, y) = x \ln y + ye^x$.
a) (2 points) What is the domain of definition of $f$?
b) (4 points) Find the unit vector $\mathbf{u}$ describing the direction of steepest ascent of $f$ at the point $(0, e)$.
c) (4 points) With $\mathbf{u}$ as in b), find the directional derivative $D_{\mathbf{u}}f(0, e)$.
d) (4 points) Find the unit vectors $\mathbf{v}$ describing the directions of zero change of $f$ at $(0, e)$, that is, the unit vectors $\mathbf{v}$ such that $D_{\mathbf{v}}f(0, e) = 0$.

3) (14 points) Find the local minima and maxima of $f(x, y) = x^4 + y^4 - 4xy + 1$.

4) (14 points) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest and farthest to $(2, 1, 3)$. For each point you find, decide if it is closest or farthest to $(2, 1, 3)$.

5) In the plane, consider the region $R$ intersection of the annulus $\{(x, y) : 1 \leq x^2 + y^2 \leq 4\}$ and the sector $\{(x, y) : 0 \leq y \leq x\}$.
a) (4 points) Sketch the region $R$.
b) (10 points) Compute the double integral $\int\int_R (1 + x + xy)dxdy$.

6) Let $R$ be the region which is the intersection of the first quadrant $\{(x, y) : x \geq 0,\ y \geq 0\}$ and the inside of the parabola described by $\{(x, y) : x \leq -4y^2 + 3\}$.
a) (4 points) Sketch the region $R$.
b) (10 points) If $f(x, y) = xy^3$, compute $\int\int_R f(x, y)dxdy$.

7) Consider the double integral $\int_0^2 \int_{y^2}^4 \frac{y^3}{x} e^{x^2} dxdy$.
a) (4 points) Sketch the region of integration.
b) (10 points) Compute $\int_0^2 \int_{y^2}^4 \frac{y^3}{x} e^{x^2} dxdy$.