

Please write your name on all of the exam booklets you use. **Show all your work** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them in detail. No calculators are permitted. Good luck!

1. (8 pts) Consider the function $f(x, y, z) = \ln xy + \ln yz + \ln xz$.
 - (a) Find the directions in which $f(x, y, z)$ increases and decreases most rapidly at $P_0(1, 1, 1)$.
 - (b) Find the derivatives of $f(x, y, z)$ in these directions.
2. (7 pts) Find parametric equations for the line tangent to the curve of intersection of the surfaces $xyz = 1$ and $x^2 + 2y^2 + 3z^2 = 6$ at the point $(1, 1, 1)$.
3. (8 pts) Find all critical points of the function $f(x, y) = x^2y + xy^2 - 3xy$ and determine their nature. Justify your answer.
4. (7 pts) Find the points on the surface $z^2 = xy + 4$ closest to the origin.
5. (8 pts) Find the volume of the solid that is bounded above by the surface $z = x^2e^{xy}$ and below by the triangular region R in the xy -plane enclosed by the lines $y = x$, $y = 0$, and $x = 1$.
6. (7 pts) Evaluate the double integral

$$\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{y}{x^2 + y^2} dy dx.$$

7. (9 pts) Let D be the region in the *first octant* that is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 9$.
 - (a) Express the volume of D as an iterated triple integral in
 - (i) spherical,
 - (ii) cylindrical, and
 - (iii) rectangular coordinates.
 - (b) Find the volume of D .
8. (6 pts) Evaluate $\int_C (x + y + z) ds$, where C is the straight line segment from $(1, 2, 3)$ to $(0, -1, 1)$.