Math 192, Prelim 1 - Solutions
September 27, 2007. 7:30-9:00

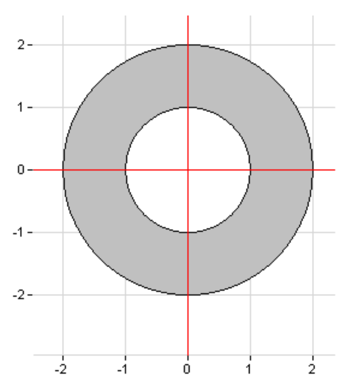
1) a) $\mathbf{v}(t) = \mathbf{i} + 2t\mathbf{j} + 2\mathbf{k}$, $\mathbf{a}(t) = 2\mathbf{j}$
b) Let $\theta(t)$ denote the angle. Then $\cos\theta(t) = \frac{\mathbf{v}(t)\cdot\mathbf{a}(t)}{|\mathbf{v}(t)||\mathbf{a}(t)|} = \frac{4t}{\sqrt{1+4t^2+4}\sqrt{4}} = \frac{2t}{\sqrt{5+4t^2}}$

2) a) $\overrightarrow{AB} = \mathbf{i} - \mathbf{j} - \mathbf{k}$ is a vector parallel to the line through A and B and $(2,0,0)$ is a point on the line so the parametric equation for the line through A and B is $x = 2+t$, $y = -t$, $z = -t$. Plug this into the equation for P_2 to get $(2+t) - 2(-t) = 0 \Rightarrow t = -\frac{2}{3}$ and use the parametrization for the line through A and B to see that the point of intersection is $(\frac{4}{3}, \frac{2}{3}, \frac{2}{3})$
b) The normals for the given planes are $\langle 1,2,3\rangle$ and $\langle 1,-2,0\rangle$ so $\langle 1,2,3\rangle \times \langle 1,-2,0\rangle = \langle 6,3,-4\rangle$ is parallel to the line of intersection. The point $(0,0,\frac{1}{3})$ is in both planes and thus on the line of intersection. Hence the line of intersection is $x = 6t$, $y = 3t$, $z = \frac{1}{3} - 4t$

3) a) $\overrightarrow{AB} = \langle -3,1,-3\rangle$, $\overrightarrow{AC} = \langle 0,1,-1\rangle$, $\overrightarrow{AB} \times \overrightarrow{AC} = \langle 2,-3,-3\rangle$ so the area of the triangle ABC, which is half the area of the parallelogram that $\overrightarrow{AB}$ and $\overrightarrow{AC}$ define, is $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}| = \frac{1}{2}\sqrt{2^2+3^2+3^2} = \frac{\sqrt{22}}{2}$
b) $\overrightarrow{BC} = \langle 3,0,2\rangle$ is a vector parallel to the line through B and C and C is definitely a point on that line. Note that $\overrightarrow{AC} \times \overrightarrow{BC} = \langle 2,3,-3\rangle$. The distance from A to the line through B and C is thus $d = \frac{|\overrightarrow{AC}\times\overrightarrow{BC}|}{|\overrightarrow{BC}|} = \frac{\sqrt{2^2+3^2+3^2}}{\sqrt{3^2+2^2}} = \frac{\sqrt{22}}{\sqrt{13}}$

4) a) $\mathbf{v_1}(t) = \cos(t)\mathbf{i} - \sin(t)\mathbf{j} + 2t\mathbf{k}$, $|\mathbf{v_1}(t)| = \sqrt{1+4t^2}$ which is not constant, $\mathbf{v_2}(t) = -3\sin(t)\mathbf{i} + 3\cos(t)\mathbf{j} + 4\mathbf{k}$ and $|\mathbf{v_2}(t)| = \sqrt{3^2+4^2} = 5$ which is constant. So particle 1 does not have constant speed but particle 2 has constant speed.
b) Note that the base point corresponds to $t = \pi$ so the arc length parameter for curve 2 is
$s(t) = \int_\pi^t |\mathbf{v_2}(\tau)|d\tau = \int_\pi^t 5d\tau = 5t - 5\pi$

5) a) We can only take square root of non-negative numbers so we either have $4 - x^2 - y^2 \geq 0$ and $x^2 + y^2 - 1 \geq 0$ or we have $4 - x^2 - y^2 \leq 0$ and $x^2 + y^2 - 1 \leq 0$. The latter option is clearly not possible because it contains points that are both inside a circle with radius 1 and outside a circle with radius 4 which clearly makes no sense. Thus if we combine the inequalities in the first option we have that points (x,y) in our domain must fulfill $1 \leq x^2 + y^2 \leq 4$. The boundary of this domain are points (x,y) that fulfill $x^2 + y^2 = 1$ or $x^2 + y^2 = 4$.



b) Closed
c) Bounded

6) a) $f(x,kx) = \frac{4kx^2}{4x^2+k^2x^2} = \frac{4k}{4+k^2}$ which varies with k so the limit does not exist by the Two Path Test.
b) If the level curves $f(x,y) = a$ and $f(x,y) = b$ intersected in the point (x_0,y_0) then we would have $f(x_0,y_0) = a$ and $f(x_0,y_0) = b$ and thus $a = b$ which is a contradiction to the fact that $a \neq b$. Thus the level curves can never intersect.

7) a) Set $x(s,t) = s - t$. Then $g(s,t) = f(x(s,t))$ and $\frac{\partial g}{\partial t} = \frac{df}{dx}\frac{\partial x}{\partial t} = f'(s-t)(-1) < 0$ for all (s,t) since $f'(s,t) > 0$ for all (s,t). So $\frac{\partial g}{\partial t}$ only takes on negative values.
b) $\frac{\partial g}{\partial s} + \frac{\partial g}{\partial t} = \frac{df}{dx}\frac{\partial x}{\partial s} + \frac{df}{dx}\frac{\partial x}{\partial t} = f'(s-t)(1) + f'(s-t)(-1) = 0$