

Please write your name on all of the exam booklets you use. **Show all your work** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them in detail. No calculators are permitted. Good luck!

1. Let  $w = x^2 + y^2$  and suppose that  $x = 2r + 3s, y = 4r - 5s$ . Calculate  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$

- (a) (5 pts) directly by writing  $w$  in terms of  $r, s$ ;  
(b) (5 pts) using the chain rule.

2. (15 pts) Find the following limits, if they exist.

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1}$

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(xy)}{xy}$ .

(c)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 - y^2}{x^4 + y^2}$ .

3. (a) (5 pts) Find an equation for the plane  $M$  through the points  $A(6, 2, 0)$ ,  $B(4, 0, 1)$  and  $C(2, 1, 1)$ .  
(b) (5 pts) Find the point on the plane  $M$  that is nearest to the origin  $(0, 0, 0)$ .

4. Consider the smooth curve

$$\mathbf{r}(t) = (t + \sin t)\mathbf{i} + (t - \sin t)\mathbf{j} + (\sqrt{2} \cos t)\mathbf{k}$$

- (a) (5 pts) Find the arc length parameter  $s$  along the curve  $\mathbf{r}(t)$  beginning from the point where  $t_0 = 0$ .  
(b) (5 pts) Let  $\mathbf{T}$  be the unit tangent vector to the given curve in the direction of the velocity vector. Find the vector  $\mathbf{T}'(s) = \frac{d\mathbf{T}}{ds}$ .

5. The plane  $y = 4$  intersects the paraboloid  $z = \frac{x^2 + y^2}{4}$  in a parabola.

- (a) (5 pts) Find an equation of the line  $L$  in the plane  $y = 4$  tangent to the parabola at  $(2, 4, 5)$ .  
(b) (5 pts) Find the angle formed by  $L$  and the positive direction of the  $x$ -axis.

OVER PLEASE —

6. Consider the function

$$f(x, y) = \begin{cases} x + y, & \text{if } x = 0 \text{ or } y = 0; \\ 1, & \text{otherwise.} \end{cases}$$

- (a) (5 pts) Find  $\frac{\partial f}{\partial x}(0, 0)$  and  $\frac{\partial f}{\partial y}(0, 0)$ .

**Hint:** Sketch a rough graph of  $f$  to help you guess their values.

- (b) (5 pts) Is  $f$  continuous at the origin? Is  $f$  differentiable at the origin? Justify your answers.