Math 192 Prelim 1, Feb. 21, 2006

Calculators are **not** allowed. Write your section number and TA's name on the front of your workbook. This exam is worth 100 points. Point values for each problem are in parentheses.

1. Let $\vec{u} = -2\vec{i} + 4\vec{j} - \vec{k}$ and $\vec{v} = 2\vec{i} + 3\vec{j} - 6\vec{k}$ be two vectors in 3-space.
a) (5) Find the angle between $\vec{u}$ and $\vec{v}$.
b) (4) Find a vector $\vec{w}$ of magnitude 5 units in the direction opposite to that of $\vec{u} + \vec{v}$.
c) (4) Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be arbitrary vectors. Answer as True or False. (No reasons needed.)
i) If $\vec{u}$ x $\vec{v} = \vec{u}$ x $\vec{w}$ and $\vec{u} \neq \vec{0}$, then $\vec{v} = \vec{w}$.
ii) If $\vec{u} \neq \vec{0}, \vec{u}$ x $\vec{v} = \vec{u}$ x $\vec{w}$ and $\vec{u} \cdot \vec{v} = \vec{u} \cdot \vec{w}$, then $\vec{v} = \vec{w}$.

2. a) (8) Find the line of intersection of the planes $2x - y + 3z = 4$ and $-x + y + 2z = 1$. Then find the plane perpendicular to this line and containing the point $(5, 6, -3)$.
b) (6) Find the distance between the two planes $2x + 3y - z = 4$ and $2x + 3y - z = 9$.

3. Let $\vec{r}(t) = (\cos t + t \sin t)\vec{i} + (\sin t - t \cos t)\vec{j}$ be the position vector of a moving particle.
a) (5) Find the length of the curve between $t = \frac{\pi}{2}$ and $t = \pi$.
b) (7) Parametrize the curve by arclength s beginning from the point where $t_o = 0$.
c) (3) Use part (b) to find the unit tangent $\vec{T}$ as a function of arclength s.

4. (10) If $\vec{r}(t) \neq \vec{0}$, then show that $\dfrac{d}{dt} |\vec{r}(t)| = \dfrac{1}{|\vec{r}(t)|} \vec{r}(t) \cdot \dfrac{d\vec{r}}{dt}$.

5. Let $z = e^{-x^2-(y-1)^2}$.
a) (6) Plot level curves for $z = 1, \dfrac{1}{e}, \dfrac{1}{e^4}$.
b) (3) For what region is this function continuous?
c) (3) What is the range of this function?

6. Let $f(x, y, z) = \dfrac{xy + yz + zx}{x^2 + y^2 - z^2}$.
a) (6) Find the domain of this function and determine whether it is open, closed, or neither, and whether it is bounded or unbounded.
b) (4) Find $\displaystyle\lim_{(x,y,z)\to(1,2,3)} f(x, y, z)$ or show it does not exist.
c) (5) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^3 + y^2}$ or show it does not exist.

7. Partial derivatives.
a) (6) The gas law for a fixed mass m of an ideal gas at absolute temperature T, pressure P, and volume V is $PV = mRT$, where R is the gas constant. Show that $\dfrac{\partial P}{\partial V}\dfrac{\partial V}{\partial T}\dfrac{\partial T}{\partial P} = -1$.
b) (7) A function $f(x, y)$ is called *harmonic* in a region D if it satisfies Laplace's equation: $\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} = 0$ throughout D. Show that $f(x, y) = \ln(x^2 + y^2)$ is harmonic everywhere in the plane except (0, 0).
c) (8) Find the equation of the line tangent to the surface $z = xy^2 \cos(x^2 y)$ at the point where $x = 2$ and $y = \pi$ and that lies in the plane $y = \pi$.