

Math 192, Prelim 1
September 27, 2007. 7:30-9:00

You are *NOT* allowed calculators, the text, or any other book or notes. *SHOW ALL WORK!*
Write your name and Lecture/Section number on each booklet you use

- 1) The position vector of a moving particle is given by $\mathbf{r}(t) = (1 + t)\mathbf{i} + (t^2 - 1)\mathbf{j} + 2t\mathbf{k}$.
 - a) (6 points) Find the velocity and acceleration vectors.
 - b) (6 points) Find, as a function of t , the cosine of the angle between the velocity and acceleration vectors.

- 2) Consider the planes P_1 and P_2 given respectively by $x + 2y + 3z - 1 = 0$ and $x - 2y = 0$.
 - a) (8 points) Find the point of intersection of P_2 with the line passing through $A(1, 1, 1)$ and $B(2, 0, 0)$.
 - b) (8 points) Find a parametric equation of the line in which the planes P_1 and P_2 intersect.

- 3) Consider the three points A, B and C with coordinates $(1, 0, 2)$, $(-2, 1, -1)$ and $(1, 1, 1)$.
 - a) (8 points) Find the area of the triangle ABC .
 - b) (8 points) Find the distance from A to the line passing through B and C .

- 4) The position vectors of two moving particles P_1, P_2 are given respectively by $\mathbf{r}_1(t) = \sin t \mathbf{i} + \cos t \mathbf{j} + t^2 \mathbf{k}$ and $\mathbf{r}_2(t) = 3 \cos t \mathbf{i} + 3 \sin t \mathbf{j} + 4t \mathbf{k}$.
 - a) (7 points) Which of these particles, if any, have constant speed? (explain your answer)
 - b) (7 points) Compute the arc length parameter of the curve described by the position vector $\mathbf{r}_2(t)$ with base point $(-3, 0, 4\pi)$.

- 5) Consider the function $f(x, y) = \sqrt{(4 - x^2 - y^2)(x^2 + y^2 - 1)}$.
 - a) (7 points) What is its domain? What is the boundary of the domain?
Include a sketch in which the domain is shaded and the boundary is indicated.
 - b) (4 points) Is the domain an open region, a closed region, neither, or both?
 - c) (3 points) Is the domain bounded or unbounded?

- 6) a) (7 points) Let $f(x, y) = \frac{4xy}{4x^2 + y^2}$. Find the limit $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ or show this limit does not exist.
b) (7 points) Let $\mathcal{C}_a, \mathcal{C}_b$ be the level curves of a function of two variables $f(x, y)$ associated with two distinct values $a \neq b$. Can \mathcal{C}_a and \mathcal{C}_b intersect? (explain your answer)

- 7) Let f be a differentiable function of one variable x whose derivative satisfies $f'(x) > 0$ for all x . Consider the function g of two variable (s, t) defined by $g(s, t) = f(s - t)$.
 - a)(7 points) Does $\frac{\partial g}{\partial t}$ take only positive values, only negative values, or does it change sign? (explain your answer)
 - b) (7 points) Compute $\frac{\partial g}{\partial s} + \frac{\partial g}{\partial t}$.