

Please write your name on all of the exam booklets you use. **Show all your work** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them in detail. No calculators are permitted. Good luck!

1. (7 pts) The position vector of a moving particle in space is given by

$$\mathbf{r}(t) = (t - \sin t)\mathbf{i} + (1 - \cos t)\mathbf{j}, \quad 0 \leq t \leq 2\pi.$$

Find the time or times in the given time interval when the velocity and acceleration vectors are orthogonal.

2. (7 pts) The temperature at a point (x, y) on a flat metal plate is given by

$$T(x, y) = \frac{60}{1 + x^2 + y^2},$$

where T is measured in $^{\circ}\text{C}$ and x, y in meters. Find the rate of change of temperature with respect to distance at the point $(2, 1)$ in

- (a) the \mathbf{i} -direction.
- (b) the \mathbf{j} -direction.

3. (10 pts) Find the following limits, if they exist.

(a) $\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + y^2}.$

(b) $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2 + y^2}.$

4. Consider the function $f(x, y) = \frac{1}{\sqrt{9 - x^2 - y^2}}.$

- (a) (6 pts) Find the domain D and the range R of f .
- (b) (2 pts) Determine if the domain of f is an open region, a closed region, or neither.
- (c) (3pts) Describe and graph the function's level curves $\{(x, y) \in D : f(x, y) = c\}$ for $c = 1/3$, $c = 1/2$ and $c = 1$.

5. (10 pts) Let $z = f(x, y)$ be a differentiable function with equal partial derivatives everywhere, that is, $f_x(a, b) = f_y(a, b)$ for all (a, b) in \mathbb{R}^2 . Consider the function

$$g(s, t) = f(s^2 - t^2, t^2 - s^2)$$

and find $\frac{\partial g}{\partial s}$ and $\frac{\partial g}{\partial t}$. What can you say about the graph of g ? Explain.

OVER PLEASE —

6. (15 pts) Consider the following three lines

$$\begin{aligned}L1 : & \quad x = 1 + t & \quad y = -2 + 3t & \quad z = 4 - t & \quad -\infty < t < \infty \\L2 : & \quad x = 2s & \quad y = 3 + s & \quad z = -3 + 4s & \quad -\infty < s < \infty \\L3 : & \quad x = -1 + 2r & \quad y = -3 + r & \quad z = 4r & \quad -\infty < r < \infty\end{aligned}$$

- (a) Show that L_1 and L_3 intersect, L_2 and L_3 are parallel, while L_1 and L_2 are skew.
- (b) Find the plane determined by L_1 and L_3 .
- (c) Find the distance between L_1 and L_2 .