PRACTICE PROBLEMS
FOR
MATH 192 PRELIM #1
(Fall 2004)

(PROBLEMS ONLY .. pages 1-4)

LINES, PLANES, ETC.

1/10

#1. Find equation(s) for the line L going through the points $P(0,1,0)$ and $Q(2,3,4)$.

#2. Consider the lines $L_1: \ x = 2+t, \ y = 2+3t, \ z = 4t$ and $L_2: \ x = 6+2s, \ y = 4+s, \ z = 0$.

a) Find the coordinates of the point of intersection of L_1 and L_2.

b) Write an equation for the plane containing L_1 and L_2.

#3.

a) Find the coordinates of the point of intersection of the line $x = 2+3t, \ y = 1+t, \ z = 4+2t$ and the plane $2x+5y+2z = 2$.

b) Find the distance from the point $(2,1,4)$ to the plane $2x+5y+2z = 2$.

#4. Find the area of the triangle with vertices P, Q, and R, if P has coordinates $(2,-3,0)$, Q has coordinates $(5,1,2)$, the vector $\overrightarrow{PR}$ is parallel to $\mathbf{i}+\mathbf{j}+\mathbf{k}$, and the vector $\overrightarrow{QR}$ is parallel to $4\mathbf{i}+8\mathbf{j}$.

#5. Express the vector $6\mathbf{i}+2\mathbf{j}$ as the sum of 2 vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a}$ is parallel to the vector $\mathbf{i}+\mathbf{j}$ and $\mathbf{b}$ is perpendicular to $\mathbf{i}+\mathbf{j}$.

#6 Find the plane that contains the point $P(1,-2,1)$ and the line:

$$\mathbf{r}(t) = t(2\mathbf{i}+4\mathbf{j}+\mathbf{k}) + (2\mathbf{i}+\mathbf{j}+3\mathbf{k})$$

#7

(a) Find the equation for the plane containing the points $(1,0,1)$ and $(0,-1,0)$ and perpendicular to the plane $x-y+z=0$.

(b) Find a vector perpendicular to the lines

$$L1: \quad x = 3, \ y = 2t, \ z = 7-t$$

$$L2: \quad x = -6+2s, \ y = 3, \ z = 3s$$

(c) Find an equation for the plane containing the lines

$$L1: \quad x = 1-t, \ y = -2+3t, \ z = -2t$$

$$L2: \quad x = 2+3s, \ y = -9s, \ z = 2+6s$$

#8

Let $P = (1,2,3)$, $Q = (2,0,4)$ and $R = (0,1,1)$.

a) (8 points) Find a normal vector to the plane of triangle PQR.

b) (7 points) Find the area of triangle PQR.

c) (7 points) Find the equation of the plane containing triangle PQR.

#9. Say $\mathbf{u}$ and $\mathbf{v}$ are vectors such that $|\mathbf{u}| > 0$ and $|\mathbf{v}| > 0$. Suppose also that $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$. What can you say about the angle between $\mathbf{u}$ and $\mathbf{v}$?

#10.

Let $P = (0, 2, 3)$ and consider the plane π given by $x + y - 2z = 4$.

(a) Find the equation of a line through P perpendicular to π.

(b) Find the point Q of intersection of this line with π.

(c) Find the distance from P to π, that is the distance from P to Q.

#11. Find the distance between the two parallel planes. (The answer is not 9.)

$$\text{Plane 1:} \quad x + 2y - 3z = 0$$
$$\text{Plane 2:} \quad x + 2y - 3z = 9$$

#12.

The following two lines are given: $L1 : x = 1 - 2t,\ y = 4t,\ z = -8 - 3t$ and $L2 : x = 4 + s,\ y = -1 - 3s,\ z = -6 + 2s$.

(a) Do the lines intersect? If yes, find the intersection point.

(b) Find the plane that contains both lines.

VECTOR-VALUED FUNCTIONS

#13. The acceleration vector of a particle in three dimensional space is $\mathbf{a}(t) = -\mathbf{i} - \mathbf{j} - 6t\,\mathbf{k}$. The velocity of the particle at time $t = 0$ is $\mathbf{v}(0) = \mathbf{0}$. The particle is at the point $(10, 2, 5)$ at time $t = 0$. Calculate the position vector $\mathbf{r}(t)$ of the particle as a function of t. Where (at what point) is the particle at time $t = 2$?

#14. The vector $\mathbf{r}(t) = 7\,\mathbf{i} - e^t\,\mathbf{j} + \sin(\sqrt{3}\,t)\,\mathbf{k}$ is the position vector of a particle at time t.

(a) Find the particle's velocity, speed, and acceleration at $t = 0$.

(b) Find the angle between the velocity and acceleration vectors at $t = 0$.

(c) For each t find a plane through the origin which contains the particle's velocity and acceleration vectors.

#15. Solve the initial value problem

$$\frac{d\mathbf{r}}{dt} = 2e^{2t}\,\mathbf{i} - (\sin t)\,\mathbf{j} - t^2\,\mathbf{k}$$
$$\mathbf{r}(0) = -\mathbf{i} + \mathbf{j} - 4\mathbf{k}$$

#16. (a) Set up and evaluate an integral for the length of that part of the curve given by $x = 2t$, $y = 4\cos \pi t$, $z = -4\sin \pi t$ between $t = 0$ and $t = 10$.

(b) Let $\mathbf{r}(t) = (\cos 2t)\mathbf{i} + (\sin 2t)\mathbf{j} + t\mathbf{k}$. Find $\mathbf{v}(t)$, $|\mathbf{v}(t)|$ and $\mathbf{T}(t)$.

#17.

A particle's velocity is given by $\mathbf{v}(t) = t\mathbf{i} + t^2\mathbf{j}$.

(a) If the particle's position at time $t = 0$ is $(2, 1)$, what is the particle's position at time $t = 1$?

(b) How far does the particle travel from time $t = 0$ to $t = 1$?

MULTIVARIABLE FUNCTIONS

#18. Find the limit if it exists: $\lim_{(x,y)\to(0,0)} y\sin(\frac{1}{x})$.

#19. Consider the function $f(x,y) = xy^2 + y\cos(x-1)$.
(a) Find the linearization at the point $(1,2)$.
(b) Find an upper bound for the magnitude $|E|$ of the error in the approximation of f by the linearization over the rectangle $|x-1| \le 0.1$, $|y-2| \le 0.1$.

#20. Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial^2 f}{\partial y^2}$, and $\frac{\partial^2 f}{\partial y \partial x}$ if $f(x,y) = xe^{2x+y} - \cos(xy)$.

#21. Let f be the function $f(x,y) = x\ln(xy)$.
a) Write the linearization of $f(x,y)$ at the point $(1,1)$.
b) Find an upper bound for the absolute value of the error in the standard linear approximation over the square $|x-1| \le 0.5$, $|y-1| \le 0.5$.

 # 22. Given $w = z\sin(xy)$ and $x = e^{t+s}$, $y = s\ln t$, $z = t^2$, calculate $\frac{\partial w}{\partial t}$ at $s = 1$, $t = 1$.

#23.
(a) Find the domain and range of the function $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-9}}$.

(b) Which of the following properties does the domain satisfy: open, closed, neither open nor closed, bounded, unbounded?

(c) Sketch the level curves $f(x,y) = \frac{1}{4}$ and $f(x,y) = \frac{1}{\sqrt{7}}$.

#24.
Find the limit if it exists:

(a) $$\lim_{(x,y)\to(2,2)} \frac{x+y+4}{\sqrt{x+y}-2}.$$

(b) $$\lim_{(x,y,z)\to(1,-1,-1)} \frac{2xy+yz}{x^2+z^2}.$$

(c) $$\lim_{(x,y)\to(0,0)} \frac{xy-x^2}{x^2+y^2}.$$

#25 Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial^2 f}{\partial x \partial y}$ for $f(x,y) = \sqrt{x^2+y^2}$.

#26. Let $f(x,y) = x^2 \sin y$.
a) (12 points) Find the linearization for $f(x,y)$ about the point $(1, \pi/2)$ and use your answer to approximate $f(.9, \pi/2 + .2)$.
b) (12 points) Find an upper bound for the error in your estimate from part a.