MATH 192 SUMMER 2007 FINAL EXAM

Please write your name on all of the exam booklets you use. ***Show all your work*** and put all your work in the exam booklet. Circle your final answers and be sure that you have explained them in detail. No calculators are permitted. Good luck!

1. (a) (6 pts) Find an equation of the plane through $A(0,0,1)$, $B(2,0,0)$ and $C(0,3,0)$.

   (b) (6 pts) Find the distance of the origin $O(0,0,0)$ from the plane of part (a).

2. Consider the ellipsoid
$$\frac{x^2}{9}+\frac{y^2}{16}+\frac{z^2}{25}=3$$

   (a) (7 pts) Find an equation for the tangent plane to the ellipsoid at the point $P(3,4,5)$.

   (b) (7 pts) The plane $x=3$ intersects the given ellipsoid in an ellipse. Find an equation of the line in the plane $x=3$ tangent to the ellipse at $P(3,4,5)$.

3. (15 pts) Let $\mathbf{F}(x,y,z)=y^2\mathbf{i}+(2xy+e^{3z})\mathbf{j}+3ye^{3z}\mathbf{k}$.

   (a) Show that $\mathbf{F}$ is conservative.

   (b) Find a function f such that $\mathbf{F}=\nabla f$.

   (c) Evaluate $\displaystyle\int_C y^2dx+(2xy+e^{3z})dy+3ye^{3z}dz$ over the line segment C from $(0,1,0)$ to $(1,1,0)$.

4. (14 pts) Find the volume of the solid that lies above the cone $z=\sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2=z$.

5. (14 pts) Use the method of Lagrange multipliers to find the points on the sphere $x^2+y^2+z^2=4$ that are closest and farthest from the point $(3,1,-1)$.

6. (14 pts) Evaluate
$$\int_C x^4dx+xydy,$$
   where C is the triangular curve consisting of the line segments from $(0,0)$ to $(1,0)$, from $(1,0)$ to $(0,1)$, and from $(0,1)$ to $(0,0)$.

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7. (14 pts) Use Stokes's theorem to evaluate

$$\int_C -y^3dx + x^3dy - z^3dz$$

where C is the positively oriented curve of intersection between the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$.

8. (14 pts) Evaluate the area of the surface of the paraboloid $z = \dfrac{x^2 + y^2}{2}$ which lies between the planes $z = 0$ and $z = 1$.

9. (14 pts) Let S be the boundary surface of the solid region D enclosed by the paraboloid $z = 1 - x^2 - y^2$ and the plane $z = 0$. Evaluate the outward flux of $\mathbf{F}(x, y, z) = y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ across S.