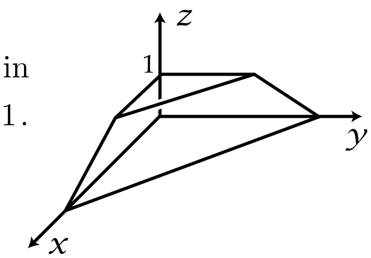


[5] **1.** (a) Write down a triple integral for the volume of the solid in the first octant bounded by the planes $x+y+z=2$ and $z=1$.

[5] (b) Compute the volume using the triple integral.

[10] **2.** Evaluate the integral $\displaystyle\int_0^1\int_{y^{1/3}}^1 \frac{1}{x^4+1}\,dx\,dy$.

[12] **3.** Evaluate the integral $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_{(x^2+y^2)^{1/4}}^{1+\sqrt{1-x^2-y^2}} 2z\tan^{-1}(y/x)\,dz\,dy\,dx$.

Bonus: Sketch the region of integration.

[5] **4.** (a) Consider a rectangular solid resting on the xy-plane with faces parallel to the coordinate planes and with its upper corners on the sphere $x^2+y^2+z^2=1$. If the corner in the first octant is at the point (a,b,c) and the solid is made from a material of variable density $\delta(x,y,z)=z$, show that the mass of the solid is $M(a,b,c)=2abc^2$.

[10] (b) Find the maximum mass that the solid in part (a) can have.

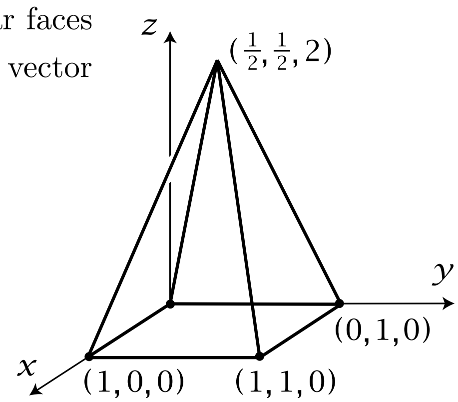
[10] **5.** Find the points on the ellipsoid $x^2+4y^2+4z^2=21$ where the tangent plane is parallel to the plane $2x+2y+z=0$.

[10] **6.** Find the tangent plane and the normal line to the surface $x^2+2y^2-z^2=8$ at the point $(1,2,-1)$.

[10] **7.** Show that all the level surfaces of the function $x^2+y^2-z^2$ meet the level surfaces of the function $x^2z^2+y^2z^2$ at right angles.

[10] **8.** Find all the critical points (x,y) of the function $f(x,y)=\sin x\sin y$ satisfying the conditions $-\pi<x<\pi$ and $-\pi<y<\pi$ and determine whether they are local minima, local maxima, or saddles.

9. Let S be the surface consisting of the four triangular faces of the pyramid shown in the figure, and let $\mathbf{F}$ be the vector field $(x^2y+z)\mathbf{i}+(2xy^2-z)\mathbf{j}+xy\sin(z^2)\mathbf{k}$.



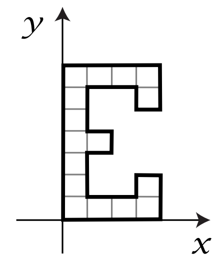
[10] (a) Compute $\iint_S(\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$ by replacing it with an equivalent line integral and computing this line integral directly. (Here $\mathbf{n}$ is the unit normal pointing outside the pyramid.)

[10] (b) Check your answer in (a) by replacing the given double integral by an equivalent double integral and computing this double integral.

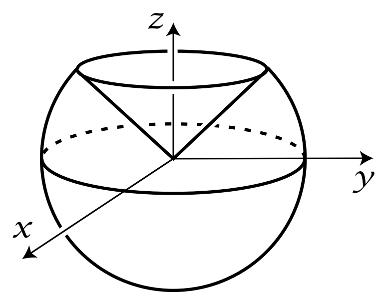
[5] **10.** (a) Find a potential function for the vector field $\big(y + \ln(1+y^2)\big)\,\mathbf{i} + \big(x + \frac{2xy}{1+y^2}\big)\,\mathbf{j}$.

[5] (b) Compute the line integral $\displaystyle\int_C \big(y + \ln(1+y^2)\big)\,dx + \big(x + \tfrac{2xy}{1+y^2}\big)\,dy$ where C is the path $\mathbf{r}(t) = t^2 \sin(\frac{\pi t}{2})\,\mathbf{i} + e^{t^2-1}\tan(\frac{\pi t}{4})\,\mathbf{j}$, $0 \le t \le 1$.

[10] **11.** For the vector field $(2xy^3 - y)\,\mathbf{i} + (2x + 3x^2y^2)\,\mathbf{j}$ compute the circulation around the closed curve C which is the boundary curve of the E-shaped region formed by 16 unit squares as shown in the figure.



[11] **12.** For the vector field $xz^2\,\mathbf{i} + yx^2\,\mathbf{j} + zy^2\,\mathbf{k}$ compute the outward flux across the boundary surface of the solid region which lies below the cone $z = \sqrt{x^2+y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 4$.



[12] **13.** Compute the flux of the vector field $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ across the portion of the surface $x^2 + y + z = 4$ with $y \ge 0$ and $z \ge 0$.