

Math 192 Final Exam, May 12, 2006

This exam consists of 10 problems on the **front and back** of this sheet. Calculators are **not** allowed. Write your section number and TA's name on the front of your workbook. This exam is worth 150 points. Point values for each problem are in parentheses. **Show all your work.**

1. (20) True/false. Write "True" or "False". Do **not** abbreviate. No reasons need be given.
 - i) For any non-zero real a and b , the vectors $a\vec{i} + b\vec{j}$ and $b\vec{i} - a\vec{j}$ are orthogonal.
 - ii) $\text{Proj}_{\vec{w}}\vec{v}$ always has the same direction as \vec{w} .
 - iii) The gradient ∇f is normal to the surface $z = f(x, y)$ at every point on the surface.
 - iv) The domain D of the function $f(x, y) = \frac{1}{x^2 - y}$ is a closed region.
 - v) The function $f(x, y) = \sin y + xe^y$ has no local or absolute extrema.
 - vi) The punctured plane $\{(x, y) : (x, y) \neq (0, 0)\}$ is a connected region but not simply connected.
 - vii) $\oint_C \frac{-ydx + xdy}{x^2 + y^2} = 0$ around every simple closed curve C containing the origin.
 - viii) $\oint_C \frac{-ydx + xdy}{x^2 + y^2} = 0$ around every simple closed curve C that does not contain the origin.
 - ix) If $f(x, y, z)$ is harmonic everywhere (i.e. $f_{xx} + f_{yy} + f_{zz} = 0$), then the outward flux of ∇f across a smooth, closed, oriented surface S is always 0.
 - x) Let \vec{F} be a vector field with continuous first partials everywhere. Let S_1 and S_2 be surfaces with unit normals n_1 and n_2 respectively and having the same boundary C . Suppose C is oriented counter-clockwise with respect to n_1 and clockwise with respect to n_2 . Then $\int_{S_1} (\nabla \times \vec{F}) \cdot \vec{n}_1 d\sigma = \int_{S_2} (\nabla \times \vec{F}) \cdot \vec{n}_2 d\sigma$.

2. (10) Find the equations of the tangent plane and the normal line to the surface $x^2 - \sin(yz^2) = 4$ at the point $(-2, \pi, 1)$. What is the equation of the plane containing the origin that is parallel to this tangent plane?

3. (12)
 - i) Find the position vector $\vec{r}(t)$ if the corresponding acceleration $\vec{a}(t) = (\cos t)\vec{i} + t^2\vec{j} + e^t\vec{k}$ and $\vec{r}(0) = \vec{i} + \vec{k}$ and the velocity $\vec{v}(0) = \vec{j} + \vec{k}$.
 - ii) Let $\vec{r}(t)$ be a position vector (not the one in part (i)) that is differentiable throughout a domain D in 3-space and has constant magnitude. Show that $\vec{r}(t)$ is orthogonal to the velocity $\vec{v}(t)$ throughout D .

4. (20) Directional Derivatives.

- i) The derivative of $f(x, y, z)$ at a given point P is greatest in the direction of the vector $\vec{v} = \vec{i} + \vec{j} + \vec{k}$. In this direction, the value of the derivative is $2\sqrt{3}$. Find the gradient vector of f at P and then find the derivative of f at P in the direction of $2\vec{i} - 3\vec{j} + 6\vec{k}$.
- ii) Given $f(x, y) = \frac{x^3}{3} + y^2$ and a unit vector $\vec{u} = u_1\vec{i} + u_2\vec{j}$, find the average value of $D_{\vec{u}}f$ over the region in the first quadrant bounded by $y = x^3$ and the lines $y = 0, x = 1$.

5. (15) If we substitute polar coordinates $x = r \cos \theta$ and $y = r \sin \theta$ in a function $z = f(x, y)$, then show that:

$$\left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial z}{\partial \theta}\right)^2 = \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$$

6. (15) Let $R = \{(x, y) : x^2 + y^2 \leq 1, y \geq 0\}$. Find the absolute maximum and minimum values of $T(x, y) = x^2 + 2y^2 - x$ on R .

7. (10) Evaluate $\oint_C \tan^{-1} y dx - \frac{xy^2}{1 + y^2} dy$, where C is the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$, oriented counter-clockwise.

8. (24) Evaluate each of these integrals.

i) $\int_0^1 \int_{2y}^2 \cos(x^2) dx dy$

ii) $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-x^2}}^{\sqrt{2-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} dz dy dx$

iii) $\int \int_S z^2 d\sigma$ where S is the sphere $\rho = a$.

9. (12) Compute the work done by the force \vec{F} around the closed curve C where $\vec{F}(x, y, z) = (z - 2y)\vec{i} + (3x - e^y)\vec{j} + (3y + \tan z)\vec{k}$ and C is the boundary of the triangle joining the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, oriented in that order, which lies in the plane $x + y + z = 1$.

10. (12) Let $\vec{F} = y\vec{i} + xy\vec{j} - z\vec{k}$ and let D be the region inside the the solid cylinder $x^2 + y^2 \leq 4$ between the plane $z = 0$ and the paraboloid $z = x^2 + y^2$. Find the outward flux of \vec{F} across the boundary of D .

GOOD LUCK IN YOUR CAREERS and HAVE A GREAT SUMMER!