Math 192, Final
December 6, 2007. 2:00-4:30

You are NOT allowed calculators, the text, or any other book or notes except for a one recto/verso letter format sheet. SHOW ALL WORK! Write your name and Lecture/Section number on each booklet you use

1) Let $f(x, y) = x^2 + y^2 - 2x$ and consider the region R in the upper half-plane $\{y \geq 0\}$ enclosed by the curves $x^2 + y^2 = 1$ and $x^2 + (y/2)^2 = 1$.
a) (4 points) Sketch the region R.
b) (12 points) Find the absolute minimum and maximum values of f in the region R and the points at which they occur.

2) (16 points) Consider the surfaces $S_1 : z = x^2 - y^2$ and $S_2 : xyz + 30 = 0$.
Find a parametric equation of the tangent line to the curve of intersection of S_1 and S_2 at the point $(-3, 2, 5)$.

3) In the plane, consider the vector field $\mathbf{F}(x, y) = e^x(1 - \cos y)\mathbf{i} + e^x(\sin y - y)\mathbf{j}$ and the region $R = \{(x, y) : 0 \leq x \leq \pi, 0 \leq y \leq \sin x\}$. Let $\mathcal{C}$ be the closed curve bounding the region R, oriented counter-clockwise.
(a) (2 points) Sketch the region R.
(b) (14 points) Evaluate the flow integral of $\mathbf{F}$ along $\mathcal{C}$.

4) (16 points) Find the largest area of a rectangle with sides parallel to the axes that can be inscribed in the ellipse $x^2 + y^2/2 = 1$.

5) (18 points) A salt storage hut has an inverted paraboloid roof $z = 4 - x^2 - y^2$ and a flat floor $z = 0$. The salt occupies half of the volume of the hut (i.e., the hut is one-half full). What is the height of the salt in the hut?

6) (18 points) Let $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + x^2\mathbf{j} + \mathbf{k}$. Evaluate the flow of $\mathbf{F}$ along the curve

$$\mathbf{r}(t) = te^{t-1}\mathbf{i} + \frac{\ln(1+t)}{\ln 2}\mathbf{j} + \sin(\pi t/2)\mathbf{k}, \quad 0 \leq t \leq 1.$$

7) Let $\mathbf{F}(x, y, z) = (x^2 + y)\mathbf{i} + (x + y)\mathbf{j} + (4y^2 - z)\mathbf{k}$ and consider the circle $\mathcal{C}$ intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $y + z = 0$, oriented counter-clockwise as viewed from above.
a)(8 points) Find curl $\mathbf{F}$.
b) (8 points) Find the circulation of the vector field $\mathbf{F}$ around $\mathcal{C}$.

8) (16 points) Consider the vector field $\mathbf{F}(x, y, z) = xy^2\mathbf{i} + x^2y\mathbf{j} + y\mathbf{k}$ and the region R enclosed by the cylinder $x^2 + y^2 = 1$ and the planes $z = -1$ and $z = 1$.
Evaluate the outward flow of $\mathbf{F}$ across the surface bounding the region R.

9) Let S be the portion of the hyperbolic cylinder $y = 1/x$ with $1 \leq x \leq 2$, $0 \leq z \leq 3$.
(a) (4 points) Sketch the surface S.
(b) (14 points) Compute the surface integral $\iint_S f d\sigma$ when $f(x, y, z) = zy^5$.