

Math 1910, Prelim 2
October 29, 2009, 7:30 PM to 9:00 PM

You are *NOT* allowed calculators, the text or any other book or notes. *SHOW ALL WORK!* Writing clearly and legibly will improve your chances of receiving the maximum credit that your solution deserves. Please label the questions in your answer booklet clearly.

Write your name and Lecture/Section number on each booklet you use. You may leave when you have finished, but if you have not handed in your exam booklet and left the room by 8:45, please remain in your seat so as not to disturb others who are still working.

1) Evaluate the following integrals.

a) (6 points) $\int_1^2 \frac{\log_2 x}{x} dx$

b) (6 points) $\int_0^{\ln 2} \frac{e^x}{(3 - e^x)^2} dx$

c) (6 points) $\int \frac{e^{2x}}{1 + e^{4x}} dx$

d) (6 points) $\int \ln^2(x) dx$

e) (6 points) $\int \frac{x + 1}{x^3 + x} dx$

2) For each of the following determine if $f(x)$ is $O(e^x)$, $O(1)$, neither, or both. Please give a short reason for each answer.

a) (5 points) $f(x) = \sinh x$.

b) (5 points) $f(x) = \tanh x$.

c) (5 points) $f(x) = \tan^{-1} x$.

d) (5 points) $f(x) = e^x(1 + \sin x)$.

3) Consider the function $y = x^x$ for $x > 0$.

a) (6 points) Calculate $\frac{dy}{dx}$.

b) (6 points) For which values of x is y increasing? For which values of x is y decreasing?

c) (6 points) Calculate $\lim_{x \rightarrow 0^+} x^x$.

(over)

4) The temperature H of a fluid in degrees Centigrade at time t minutes is given by the function

$$H(t) = \left[\frac{1}{\pi} \tan^{-1}(t) + \frac{t}{1+t} \right] 100^\circ C$$

on the domain $t \geq 0$. So $H(1) = 75^\circ C$.

- a) (7 points) Show that $H(t)$ is a one-to-one function.
- b) (7 points) Calculate $(H^{-1})'(75^\circ C)$.

5) The size of a population $y(t)$ evolves according to the differential equation

$$\frac{dy}{dt} = -2y, \quad y(0) = y_0.$$

- a) (6 points) Write an expression for $y(t)$.
- b) (6 points) What happens to the size of the population as $t \rightarrow \infty$? Does your answer depend on the initial size y_0 ?
- c) (6 points) How long does it take for the size of the population to become a quarter of its starting size? Does your answer depend on the initial size y_0 ?