1. a. Let $u = 1 + \ln\sqrt{x}$. Take a derivative w.r.t x to get $du = \frac{dx}{2x}$. By our substitution, we have $\int 2u du = u^2 + C = (1 + \ln\sqrt{x})^2 + C$

   b. Use the Logarithmic Derivative. Firstly, take ln on both sides to get $\ln y = \frac{\ln x}{\ln 2}\ln x = \frac{(\ln x)^2}{\ln 2}$. Take a derivative w.r.t. x to get $\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x \ln 2}$. Multiplying by y, we have $y' = \frac{2}{x}\log_2 x \cdot x^{\log_2 x}$.

2. a. True: $\frac{e^{-x}x^3 + 5x^2}{x^2} = \frac{x}{e^x} + 5 < 6$ for large x.

   b. False: $\lim_{x\to\infty} \frac{\frac{2}{x}}{\frac{1}{x} + \frac{\ln x}{x^2}} = \lim_{x\to\infty} \frac{2}{1 + \frac{\ln x}{x}} = 2 \neq 0$

3. a. $\alpha = \cos^{-1}(-\frac{1}{2}) - \frac{\pi}{2} = \frac{2\pi}{3} - \frac{\pi}{2} = \frac{\pi}{6}$. Hence $\sin\alpha = \sin\frac{\pi}{6} = \frac{1}{2}$, $\cos\alpha = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$.

   b. $\alpha = \sin^{-1}\left(\frac{2}{\sqrt{x^2+4}}\right)$ indicates the diagram $\Rightarrow \tan\left(\sin^{-1}\left(\frac{2}{\sqrt{x^2+4}}\right)\right) = \tan\alpha = \frac{2}{|x|}$

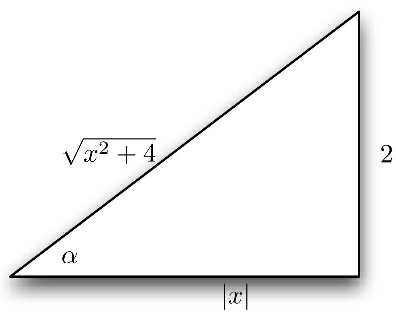


Figure 1: Problem 3: b)

Problem 4. (16 pts) Consider $f(x) = 2 + x + x^3$.

a) (8 pts) Find the largest domain, on which we can define f^{-1}, the inverse of f. Explain why.

$f'(x) = 1 + 3x^2 > 0$ for all $x \in \mathbb{R}$

Thus the function $f(x) = 2 + x + x^3$ always increases, therefore "1-to-1" for all $x \in \mathbb{R}$, with range of f being $\mathbb{R}$ as well.

So we can define f^{-1} on $\mathbb{R}$.

b) (8 pts) Find the value of $\frac{d}{dx} f^{-1}(x)$ at $x = 2$.

$$\left.\frac{d}{dx} f^{-1}(x)\right|_{x=2} = \frac{1}{\left.\frac{d}{dx} f(x)\right|_{x=f^{-1}(2)}} = \frac{1}{\left.\frac{d}{dx} f(x)\right|_{x=0}} = \frac{1}{\left.(1+3x^2)\right|_{x=0}}$$

$$= \boxed{1}$$

$f(a) = 2 + a + a^3 = 2$

$\Longrightarrow a + a^3 = 0$

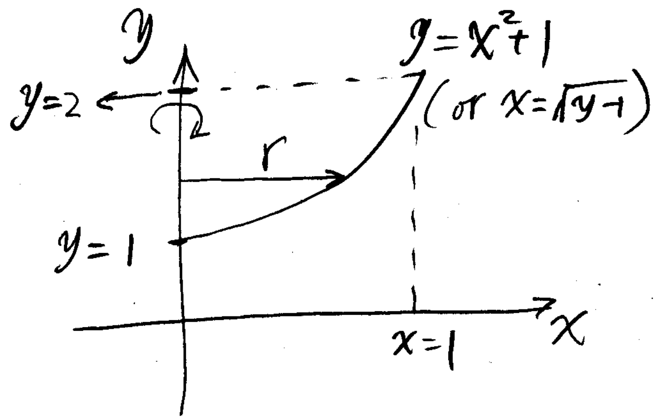
$a(1 + a^2) = 0$

$\Longrightarrow a = 0$

Thus, $f(0) = 2$ and $f^{-1}(2) = 0$

Problem 5. (20 pts) Consider the curve $y = x^2 + 1$, $0 \le x \le 1$.

a) (10 pts) Evaluate the area of the surface generated by revolving the curve about the y-axis.



$$r = x = \sqrt{y-1}$$

$$\frac{dx}{dy} = \frac{1}{2\sqrt{y-1}}$$

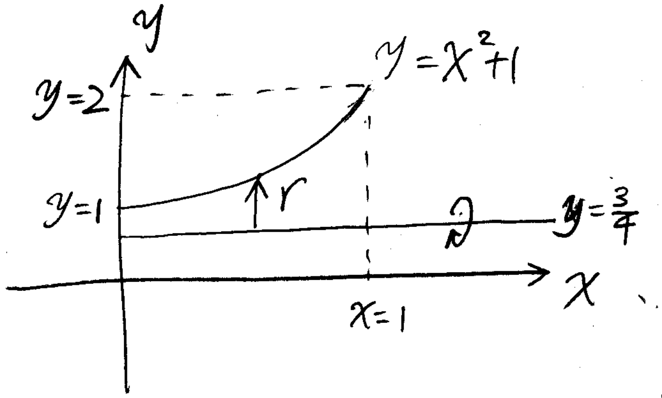
$$\sqrt{1+\left(\frac{dx}{dy}\right)^2} = \sqrt{1+\frac{1}{4(y-1)}}$$

$$S = \int_1^2 2\pi r \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 2\pi\sqrt{y-1} \cdot \sqrt{1+\frac{1}{4(y-1)}}\, dy$$

$$= \boxed{\frac{\pi}{6}\left(5^{3/2}-1\right)}$$

b) (10 pts) Evaluate the area of the surface generated by revolving the curve about the horizontal line $y = \frac{3}{4}$.



$$r = (x^2+1) - \frac{3}{4} = x^2 + \frac{1}{4}$$

$$\frac{dy}{dx} = 2x$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+4x^2}$$

$$S = \int_0^1 2\pi r \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^1 2\pi \left(x^2+\frac{1}{4}\right)\sqrt{1+4x^2}\, dx$$

$$= \frac{\pi}{2}\left(\frac{13}{8}\sqrt{5} - \frac{3}{16}\ln\left(-2+\sqrt{5}\right)\right)$$