SUPPLEMENTARY ASSIGNMENT 1

DUE TH OCTOBER 29, 2009

(Allen's Favorite Rule of Thumb Counterexample)

In homework problem 36d of chapter 16, you contemplated 1000 plays of a slot machine, where on average one play resulted in a \$.08 casino profit with a standard deviation of \$120. Since $1000 \gg 40$, you then "correctly" used the CLT while thinking about $N(.08, \frac{120}{\sqrt{1000}})$ to conclude that there was still a 49% chance the casino would lose money on that machine on a given day.

Strike you as counterintuitive? This problem will show you that at least sometimes, the calculation in 36d is not truly applicable.

In particular, let's assume the slot machine is "jackpot or nothing." Meaning, when you put your \$1 in, most of the time (i.e. with probability $1-p$) you just lose, but with probability p, you get a jackpot of amount A. Let's see what A and p should be to approximate the average \$.92 gross payout with a standard deviation of \$120.

Our probability model for the gross return (the RV Y, say) to the gambler for one play is

Amount	Prob.
0	1-p
A	p

Note $Y = AX$, where X is Bernoulli(p), so the mean and standard deviation of Y are quite clear. (The net return which keeps track of overall loss is $Y - 1$.)

Problem 1. For the RV Y described above, complete the following table for its mean and standard deviation:

p	A	mean of Y	st. dev. of Y
.92	1	.92	.271
.04	600	24	117.58
.092	10	?	?
.00092	1000	?	?
.0000092	100000	?	?

Problem 2. The mean of Y is Ap. And when p is small, the standard deviation of Y is close to $A\sqrt{p}$. Using this observation find values of A and p for which the mean and standard deviation of Y are within 1% of the values $\mu = .92$ and $\sigma = 120$ given in problem 36.

Problem 3. With your value of p from problem 2, what is the probability that no jackpots are given out in 1000 plays of the slot machine? Your answer should be wildly different from the $51\% = 100\% - 49\%$ that normal approximation in problem 36d gave. Can you give a simple explanation of why normal approximation of the RV Y should not be expected to be accurate?