Math 1220 Quiz 2
October 14, 2009

Name SOLUTIONS

(3 pts) (1) Suppose a solid has height h. Its horizontal cross-section at height z feet above its base is a rectangle with length $3+2z$ and width $7-2z$ feet. Write an integral that gives the volume of the solid. DO NOT INTEGRATE!

Height – 0 to h

Cross-section area at height z : $(7-2z)(3+2z)$

$$\text{Volume} = \int_0^h (7-2z)(3+2z)\,dz$$

(4 pts) (2) Set up, but do not integrate, an integral expressing the surface area of the solid generated by rotating the portion of the curve $y = x^2 - 2\ln x$ from $x=1$ to $x=e$ around the x-axis.

We are rotating around the x-axis, so

$$r = |x^2 - 2\ln x| = (x^2 - 2\ln x) \quad \text{for } 1 \le x \le e$$

$$ds = \sqrt{1+(y')^2}\,dx = \sqrt{1+\left(2x-\tfrac{2}{x}\right)^2}\,dx$$

$$\text{Surface area} = \int_1^e |x^2 - 2\ln x| \cdot \sqrt{1+\left(2x-\tfrac{2}{x}\right)^2}\,dx$$

$$= \int_1^e (x^2 - 2\ln x) \cdot \sqrt{1+\left(2x-\tfrac{2}{x}\right)^2}\,dx$$

OVER →

(3) Suppose a parametric curve is given by the equations $x = \frac{1}{t} + 1$ and $y = t^2 + 2t$, for $0 < t < 2$.

(3pts) a) Eliminate the parameter to give a Cartesian equation in x and y whose graph contains the parametric curve.

$$x = \frac{1}{t} + 1 \implies x - 1 = \frac{1}{t} \implies t = \frac{1}{x-1}$$

$$\text{so } y = \left(\frac{1}{x-1}\right)^2 + \frac{2}{x-1} = \frac{1}{(x-1)^2} + \frac{2}{x-1}$$

$$= \frac{2x-1}{(x-1)^2}$$

(3pts) b) Specify the domain for the Cartesian equation you gave in part (a) which gives the parametric curve for the specified values of t.

We know $\frac{1}{t} + 1$ is decreasing for positive t, so the smallest value of x will be at $t = 2$.

$$\boxed{\text{Domain: } \left(\frac{3}{2}, \infty\right)} \quad \text{or} \quad x > \frac{3}{2}$$

as $x(2) = 3/2$ and $\lim_{t \to 0} x(t) = \infty$.

(2pts) c) Describe the direction of the parametric curve, i.e. as t increases, is the curve moving to the left or the right (with respect to the x-axis)? A graph of the curve is sufficient, but not necessary, in answer to this question.

It is moving to the left (and up)