Calculators, books, and notes are not permitted. Show all your work.

(20 pts) **1.** Compute the surface area of the surface of revolution obtained by rotating the curve $y = \ln x$, $0 < x \leq 1$, about the y-axis.

(25 pts) **2.** Consider the curve parametrized by $x = \sin 2t$, $y = \sin t$ for $0 \leq t \leq 2\pi$.

(a) Find the slope(s) of the curve where it passes through the origin.

(b) Find the points on the curve where the tangent lines are horizontal and vertical.

(c) Sketch the curve using the information in parts (a) and (b). You do not need to worry about concavity.

(d) Set up completely, but do not evaluate, an integral for the arclength of the curve.

(25 pts) **3.** (a) Sketch the polar curve $r = 2 + \cos 4\theta$.

(b) Compute the area of the region inside this curve.

(c) Compute the area of the portion of the region in part (b) that lies outside the circle $r = 2$.

(20 pts) **4.** Compute the surface area of the surface of revolution obtained by rotating the lemniscate $r^2 = \cos 2\theta$ about the x-axis.

(10 pts) **5.** The curve $r = \pm \sin n\theta$ is a flower with $2n$ petals, inscribed in a circle of radius 1. What proportion of the area of the circle lies inside the $2n$ petals? (Compute the area inside the petals and compare this with the area of the circle.)

Some formulas:

$$\cos^2 x = \frac{1 + \cos 2x}{2} \qquad \sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\int \ln x \, dx = x \ln x - x + C \qquad \int \tan x \, dx = \ln|\sec x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C$$