Calculators, books, and notes are not permitted. Show all your work.

(30 pts) **1.** Compute the following integrals:

(a) $\displaystyle\int \frac{x^4+3x^3+3x^2+x+1}{x^4+x^2}\,dx$

(b) $\displaystyle\int_0^2 \frac{dx}{(4+x^2)^{5/2}}$ (Note the limits of integration.)

(c) $\displaystyle\int \ln\sqrt{1+x^2}\,dx$

(15 pts) **2.** (a) Sketch the curve $r^2 = \cos 2\theta$ in polar coordinates and compute the area inside the curve.

(b) Compute the arclength of the curve parametrized by $x = \cos^3 t$, $y = \sin^3 t$, for $0 \le t \le 2\pi$.

(10 pts) **3.** Consider the region between the curve $y = \sin x$ and the x-axis for $0 \le x \le 2\pi$. Compute the volume of the solid obtained by rotating this region about the y-axis. (Be careful: there is something slightly tricky about this problem.)

(15 pts) **4.** Determine whether each of the following series converges:

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{2n^2+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

(d) Determine the values of $x > 0$ for which the following series converges:

$$\sum_{n=1}^{\infty} \frac{1}{x^n + x^{n-1} + \cdots + x + 1}$$

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(20 pts) **5.** (a) Determine the values of x for which the series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x-1)^{2n}}{n^2}$ converges.

(b) Compute the Maclaurin series for the function $f(x) = \dfrac{x^2}{(1-x)^2}$.

(c) Find the Taylor series $\displaystyle\sum_{n=0}^{\infty} a_n(x-2)^n$ for $\ln x$ and determine the interval of convergence.

(d) Suppose the ratio test yields the information that a certain series $\displaystyle\sum_{n-0}^{\infty} a_n(x-c)^n$ has radius of convergence equal to R. Show that the derivative series $\displaystyle\sum_{n-0}^{\infty} na_n(x-c)^{n-1}$ has the same radius of convergence R.

(10 pts) **6.** (a) Find the general solution of the differential equation $x^2y' - y = 1$.

(b) Find the solution of $y'' + 2y' + 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = 0$.

(c) Find a differential equation satisfied by all of the hyperbolas $xy = C$, for C an arbitrary constant.

(d) Find a family of curves, each of which intersects each hyperbola in part (c) at right angles.

Some formulas:

$\cos^2 x = \dfrac{1+\cos 2x}{2}$ $\qquad$ $\sin^2 x = \dfrac{1-\cos 2x}{2}$

$\displaystyle\int \ln x\,dx = x\ln x - x + C$ $\qquad$ $\displaystyle\int \tan x\,dx = \ln|\sec x| + C$

$\displaystyle\int \sec x\,dx = \ln|\sec x + \tan x| + C$

$\displaystyle\int \sec^3 x\,dx = \frac{1}{2}\sec x\tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$